The Math Behind the Weather

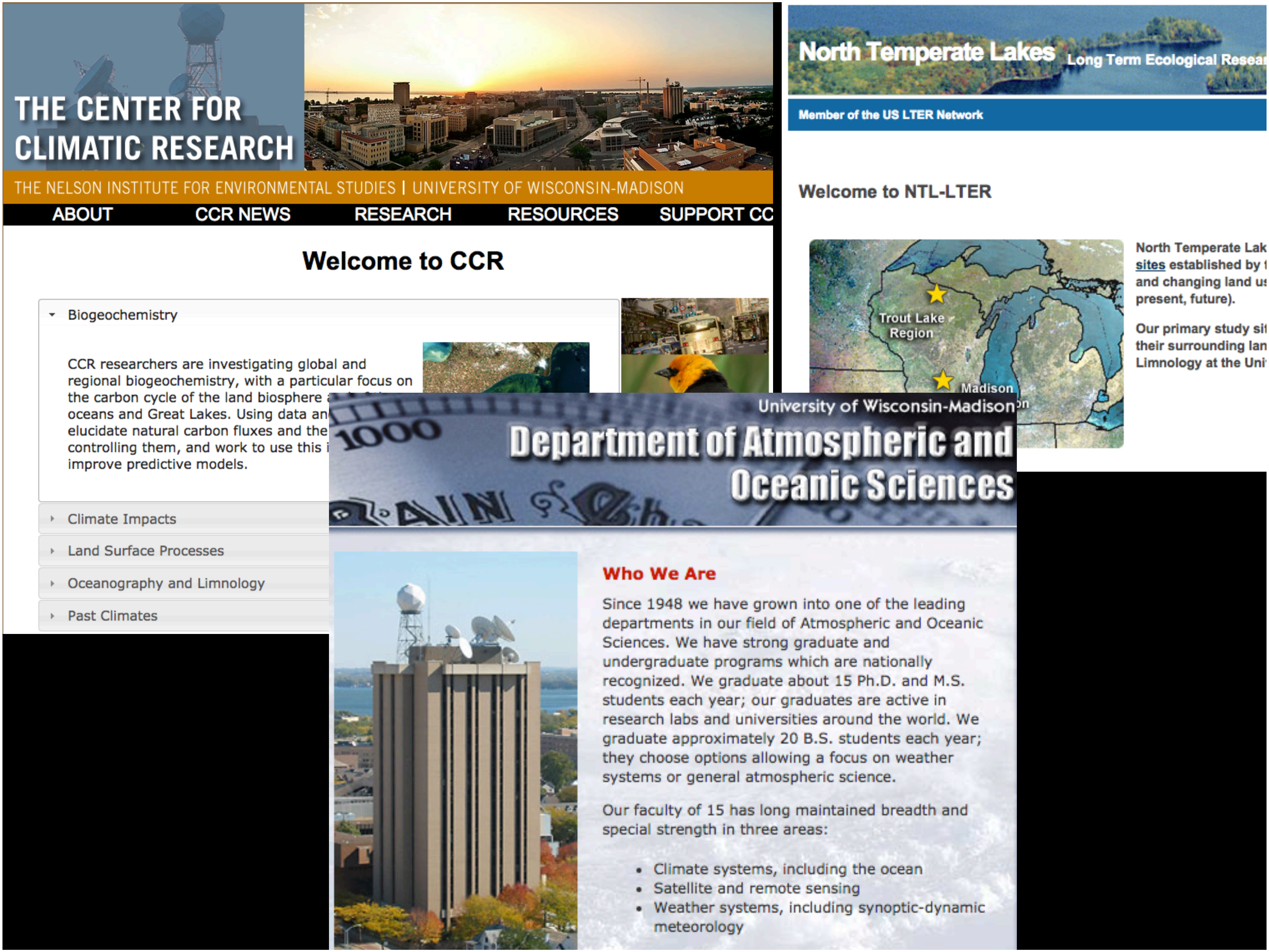
Ankur Desai, Dept of Atmospheric and Oceanic Sciences
UW-Madison, 15 March 2018

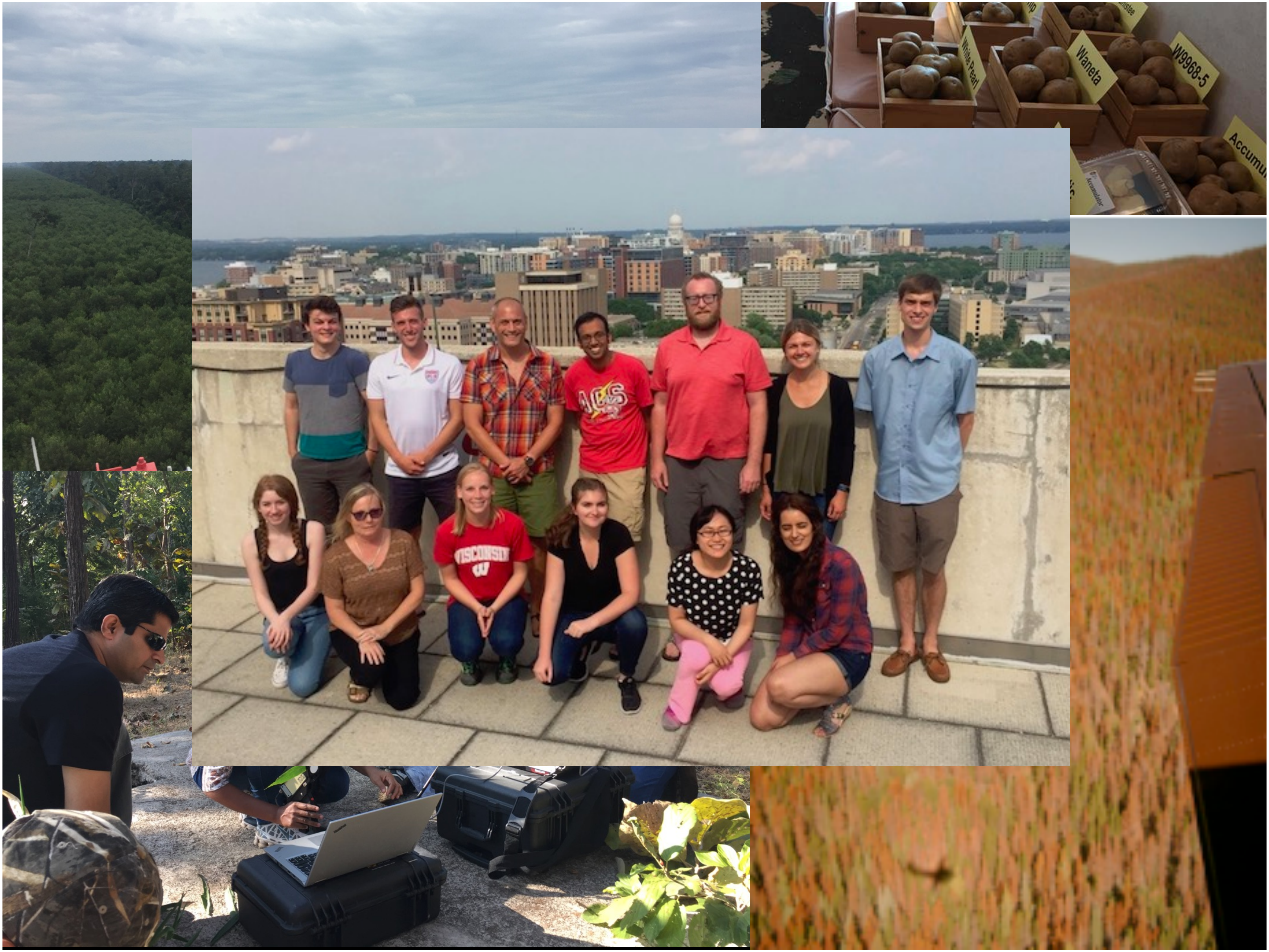
Columbia Pictures

http://flux.aos.wisc.edu desai@aos.wisc.edu @profdesai

Who Am I?

- I was born and raised in New Jersey
- I live in Madison with my wife and three daughters
- I am a climate scientist who has spent that past 2 decades studying how plants, climate, and weather all influence each other


THE CENTER FOR CLIMATIC RESEARCH
THE NELSON INSTITUTE FOR ENVIRONMENTAL STUDIES | UNIVERSITY OF WISCONSIN-MADISON
ABOUT
CCR NEWS
RESEARCH
RESOURCES
SUPPORT CC
Welcome to CCR
Biogeochemistry
CCR researchers are investigating global and regional biogeochemistry, with a particular focus on the carbon cycle of the land biosphere
oceans and Great Lakes. Using data an
elucidate natural carbon fluxes and the
controlling them, and work to use this i
improve predictive models.
Climate Impacts
Land Surface Processes
Oceanography and Limnology
Past Climates
North Temperate Lakes
Long Term Ecological Resear
Member of the US LTER Network
Welcome to NTL-LTER
Trout Lake Region
Madison
North Temperate Lak
sites established by
and changing land us
present, future).
Our primary study si
their surrounding lan
Limnology at the Uni
University of Wisconsin-Madison
Department of Atmospheric and Oceanic Sciences
Who We Are
Since 1948 we have grown into one of the leading departments in our field of Atmospheric and Oceanic Sciences. We have strong graduate and undergraduate programs which are nationally recognized. We graduate about 15 Ph.D. and M.S. students each year; our graduates are active in research labs and universities around the world. We graduate approximately 20 B.S. students each year; they choose options allowing a focus on weather systems or general atmospheric science.
Our faculty of 15 has long maintained breadth and special strength in three areas:
• Climate systems, including the ocean
• Satellite and remote sensing
• Weather systems, including synoptic-dynamic meteorology


White Pearl
Waneta
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WISCONSIN


ICE
SNOW
FREEZ
SLEET
SNOW
COLD
P.C. VEY

NATIONAL WEATHER SERVICE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

HOME | FORECAST | PAST WEATHER | SAFETY | INFORMATION | EDUCATION | NEWS | SEARCH | ABOUT

Go

View Location Examples

Your local forecast office is
Milwaukee, WI

News Headlines

- NWS Milwaukee Student Volunteer Opportunity
- Fox River-Southeast Wisconsin Flood Inundation Mapping-New Service
- Seeking CoCoRahs Volunteers
- Severe Weather Safety/Spotter Training: Thursday, March 15th-Portage 6:30pm

Additional Headlines

En Español | Share |

Current conditions at
Madison, Dane County Regional-Truax Field (KMSN)
Lat: 43.14°N **Lon:** 89.35°W **Elev:** 860ft.

Fair
25°F
-4°C

Humidity	88%
Wind Speed	Calm
Barometer	29.87 in (1012.3 mb)
Dewpoint	22°F (-6°C)
Visibility	7.00 mi
Last update	15 Mar 7:53 am CDT

More Information:
Local Forecast Office
More Local Wx
3 Day History
Mobile Weather
Hourly Weather Forecast

Extended Forecast for
Madison WI

Today	Tonight	Friday	Friday Night	Saturday	Saturday Night	Sunday	Sunday Night	Monday
			20%					20%
Sunny	Clear	Sunny	Slight Chance Snow	Mostly Sunny	Mostly Clear	Sunny	Partly Cloudy	Mostly Cloudy then Slight Chance Rain
High: 43 °F	Low: 19 °F	High: 40 °F	Low: 24 °F	High: 46 °F	Low: 27 °F	High: 50 °F	Low: 30 °F	High: 42 °F

Friday and Saturday...Forecast confidence is medium.

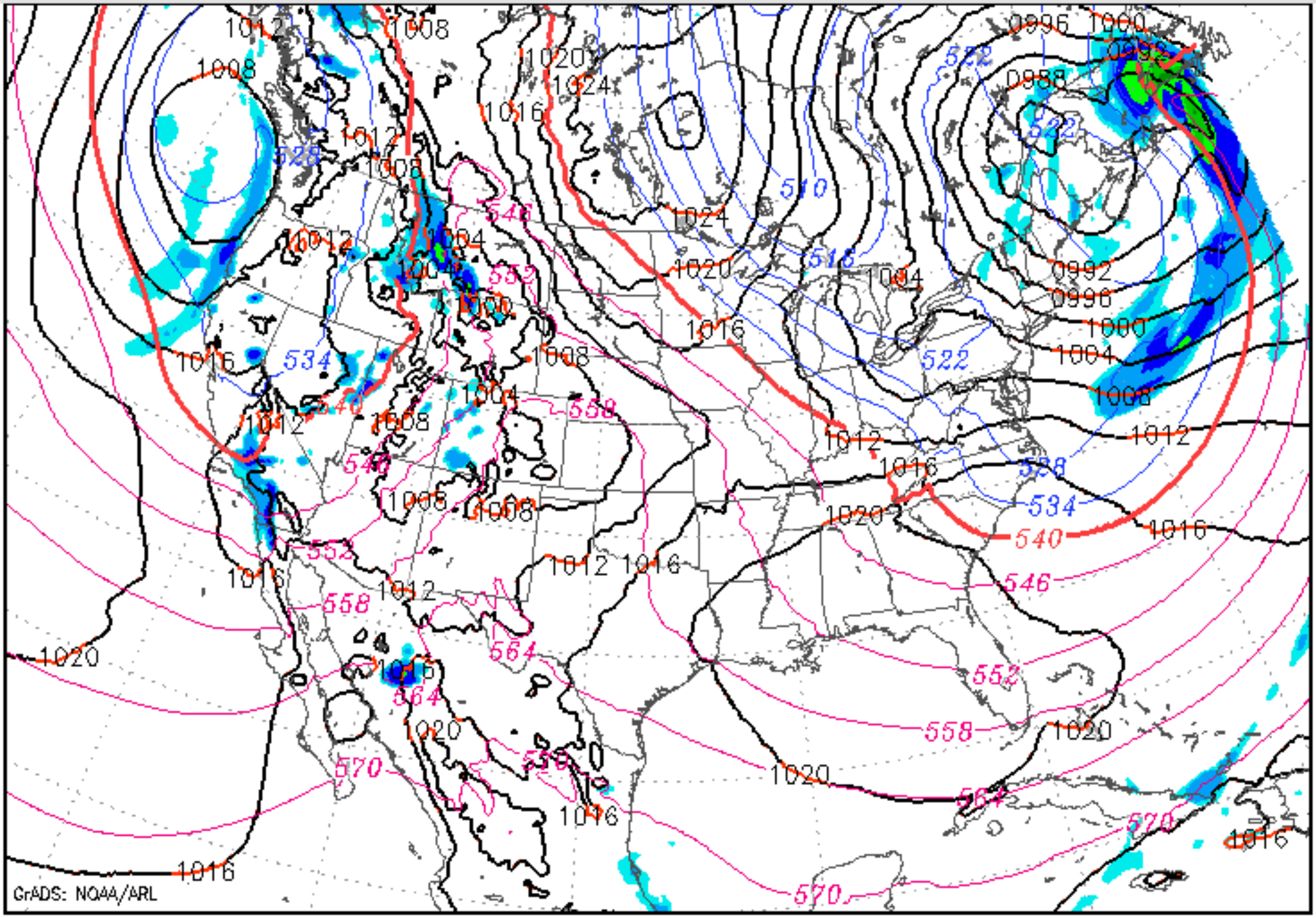
A **shortwave** **trough** will lose **amplitude** while progressing from the central High Plains to the Ohio River valley. Rapid **cyclogenesis** will commence along the Colorado **Front** Range on Friday, with the occluding surface **cyclone** reaching the mid Mississippi and Ohio Valleys by late Saturday. Forecast models are in good agreement with keeping the low, and its associated tight **baroclinic zone**, to our south. However, there is a window Friday night where column **moisture** increases (**Precipitable Water** values ~ 0.5" in southwest Wisconsin), juxtaposed with robust lift and modest **moisture** within the **dendritic** (snow) growth zone. The **GFS** **ensemble** members are now in better agreement with the surface low track, with most solutions bringing measurable precipitation into southwest Wisconsin on Friday night. This compares favorably with the latest **ECMWF** and Canadian deterministic runs, which suggest the same. The **NAM** appears a rather wet outlier (with > 0.25" of **QPF** southwest of Madison) and is discounted for now. All of this is to say that we`re seeing enough model consistency to increase rain/snow chances on Friday night, particularly for areas southwest of Madison and towards the Illinois border where light snow accumulations are possible.

Sunday through Thursday...Forecast confidence is medium.

Shortwave troughing ejects out of the Four Corers region on Sunday, before reaching the lower Missouri and Ohio Valleys by Monday night. The associated surface low passes through the mid Mississippi

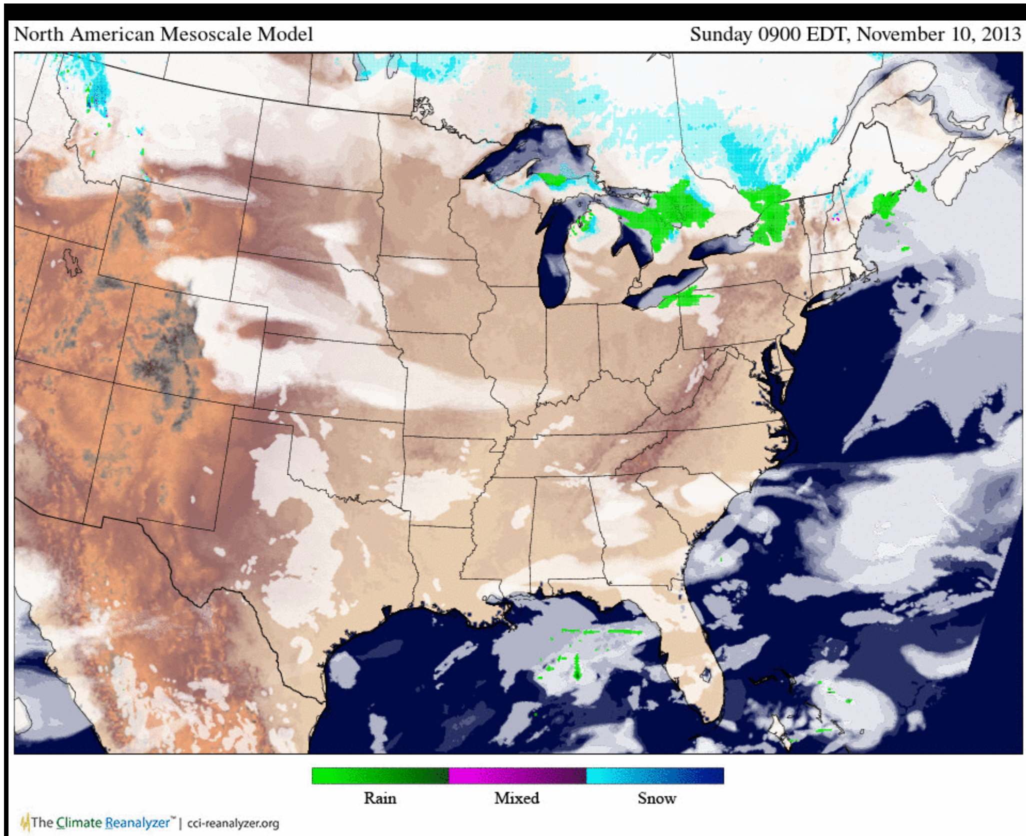
https://forecast.weather.gov/product.php?site=MKX&issuedby=MKX&product=AFD&format=CI&version=1&glossary=1

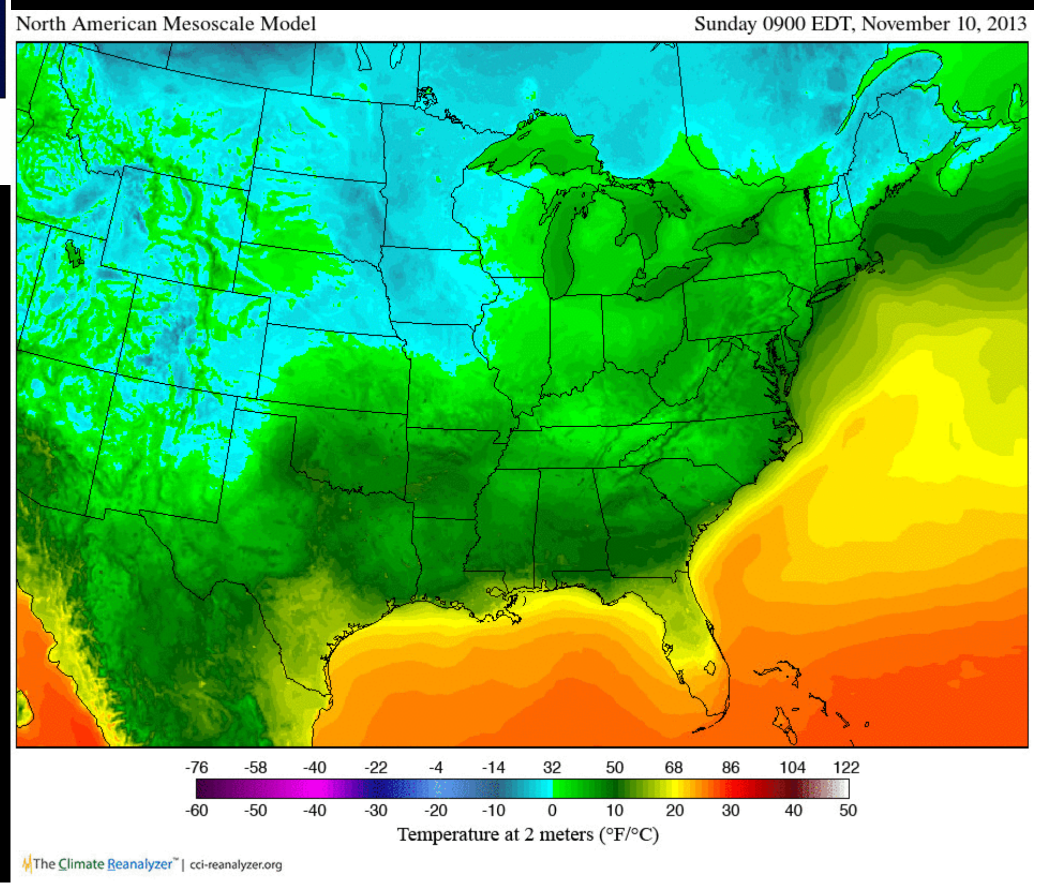
NAM MSLP, 1000–500mb Thickness, & Precipitation



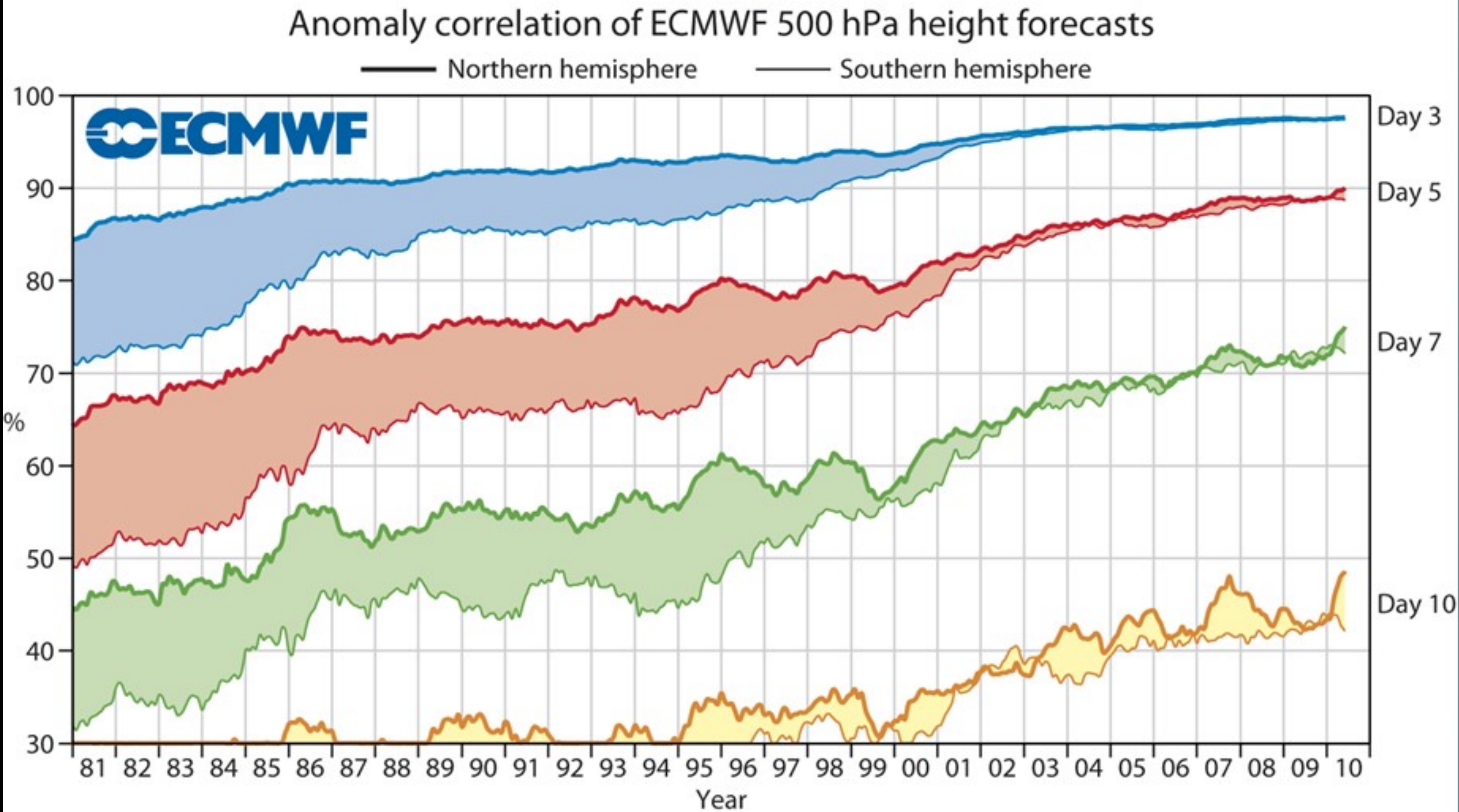
Analysis: 12Z Thu 15 MAR, 2018

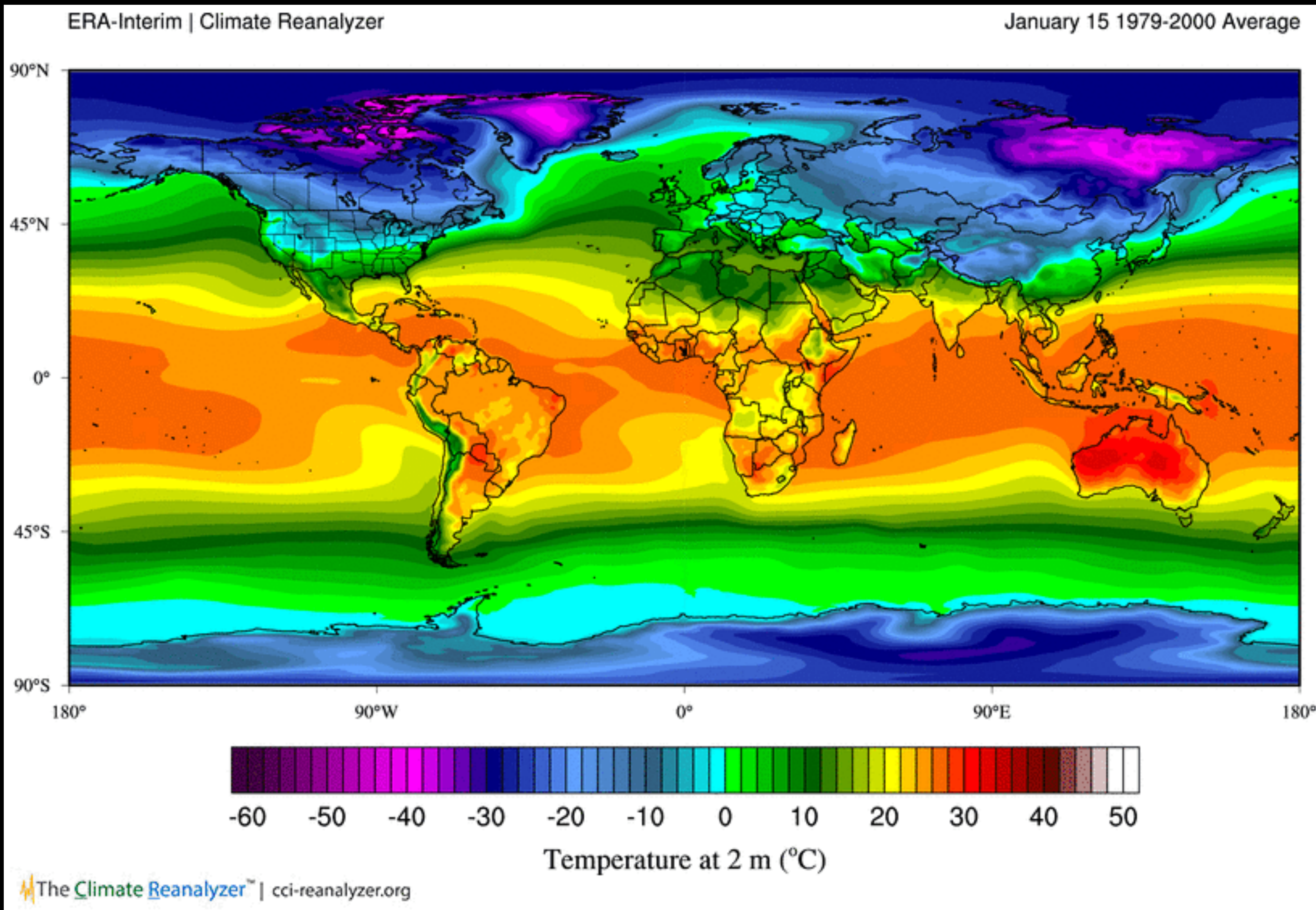
SLP *(mb–1000)*, 1000–500mb Thk *(dam)*, 3hr Precip.*(mm)*


North American Mesoscale Model
Sunday 0900 EDT, November 10, 2013
Rain
Mixed
Snow
The Climate Reanalyzer™ | cci-reanalyzer.org

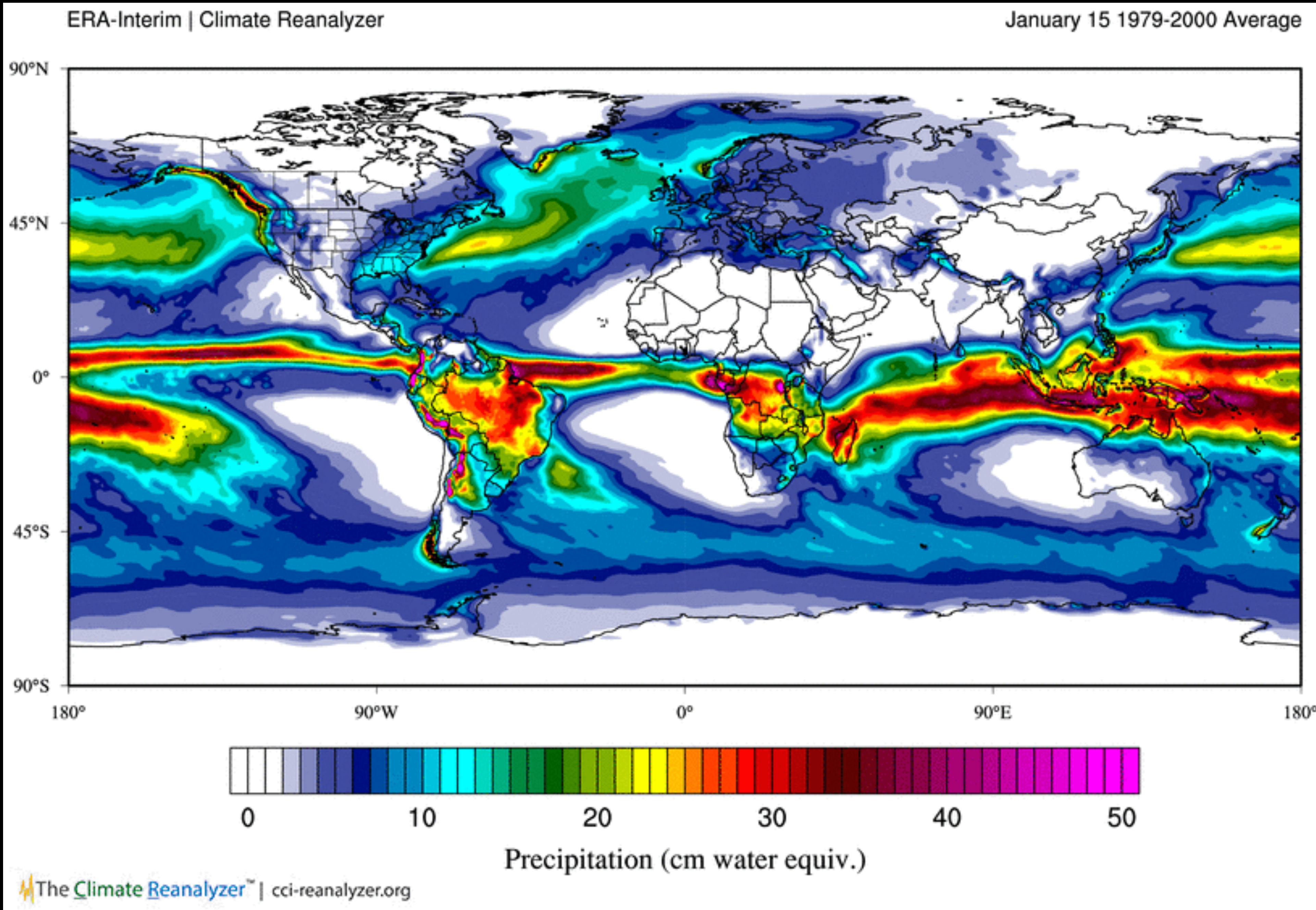

North American Mesoscale Model
Sunday 0900 EDT, November 10, 2013
-76 -58 -40 -22 -4 -14 32 50 68 86 104 122
-60 -50 -40 -30 -20 -10 0 10 20 30 40 50
Temperature at 2 meters (°F/°C)
The Climate Reanalyzer™ | cci-reanalyzer.org

Advances in Global and Regional Weather Forecasts

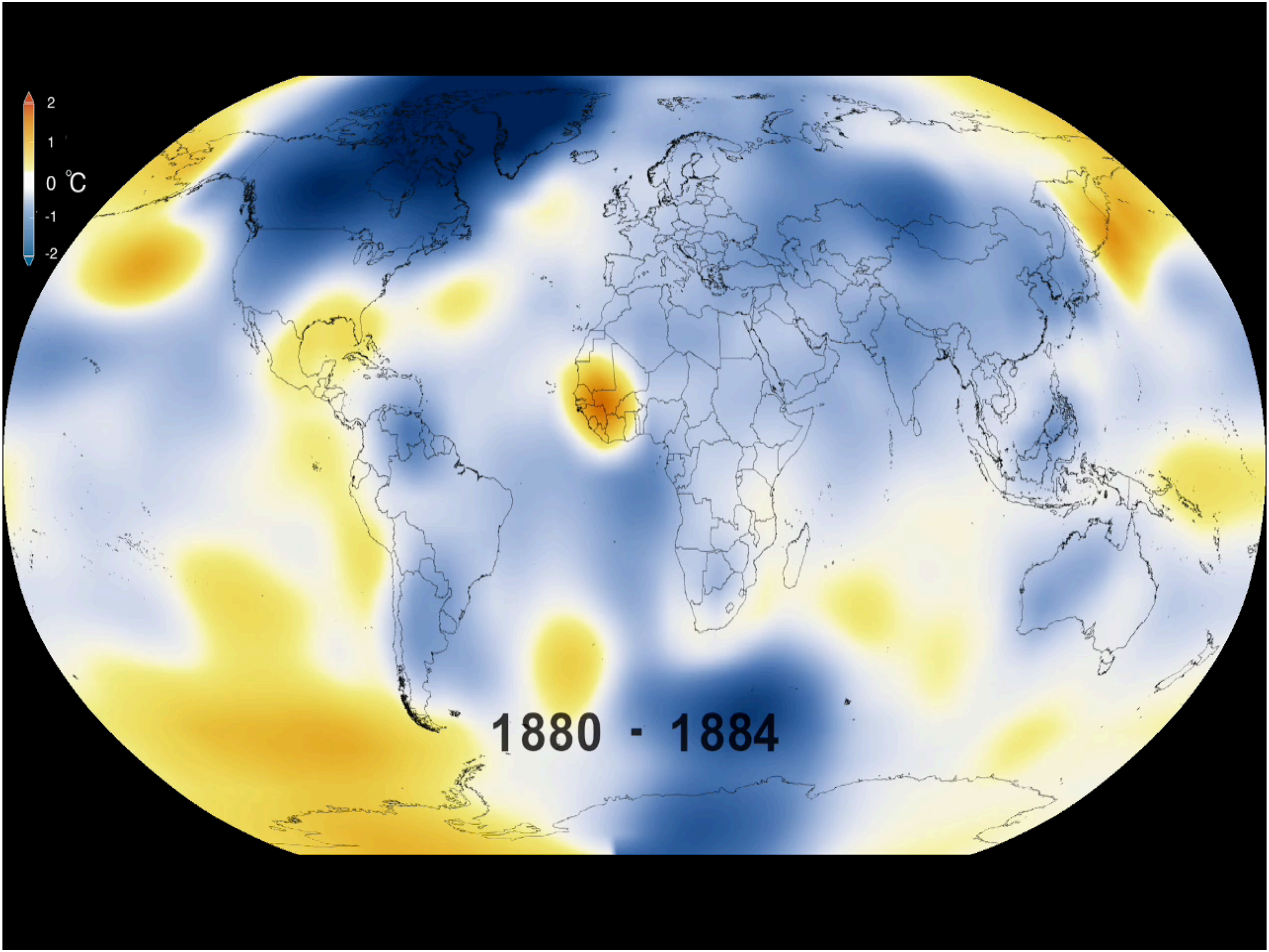


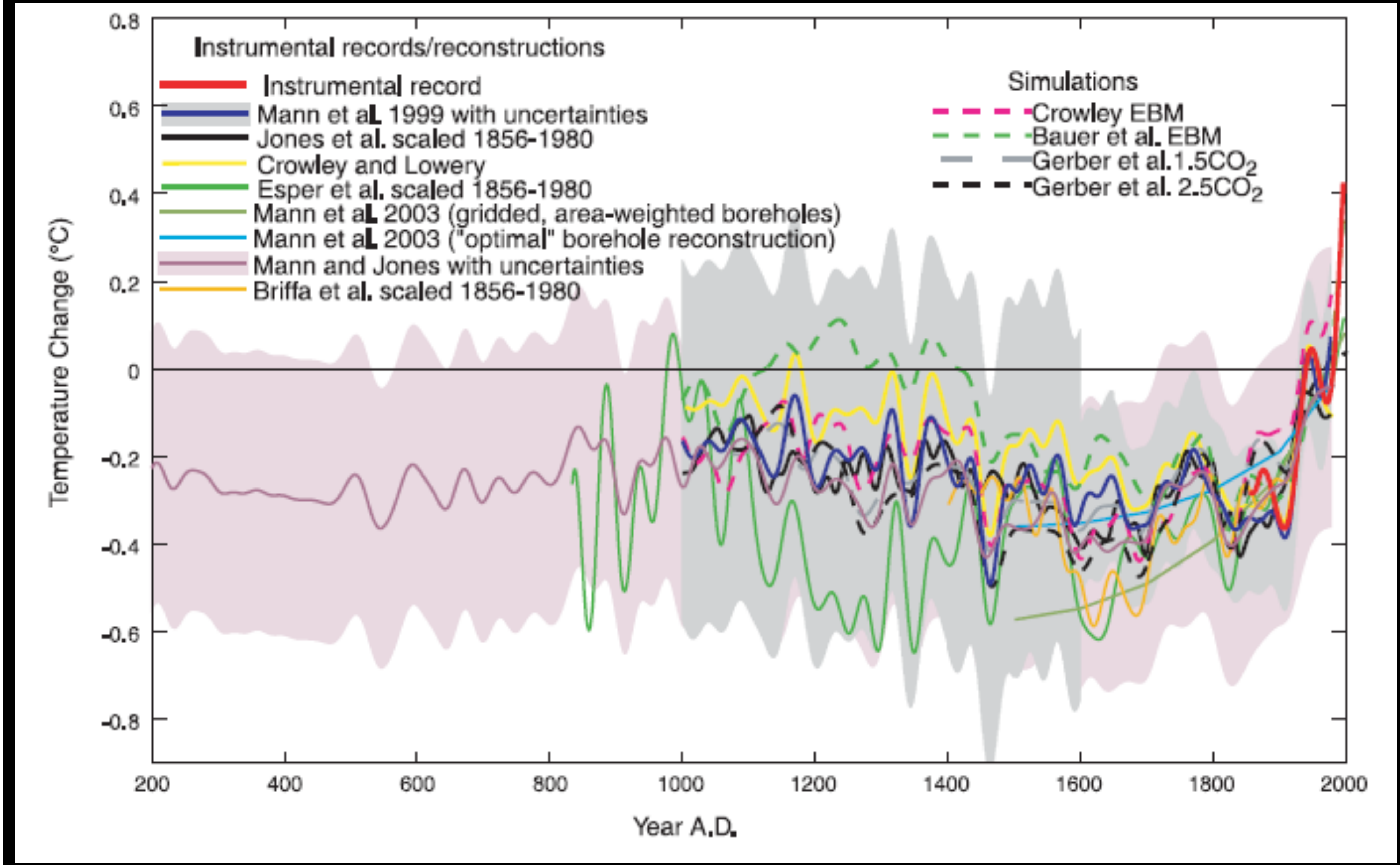


http://cci-reanalyzer.org/



http://cci-reanalyzer.org/


2
1
0 °C
-1
-2
1880 - 1884



Mann et al., 2003, EOS

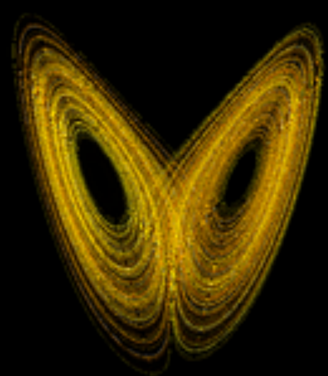
Climate Model = Weather Model

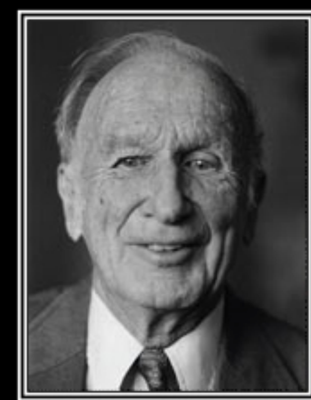
Major math innovations related to meteorology

- Navier-Stokes equation in turbulent, rotating reference frame
- Chaos in Non-linear dynamical systems
- Numerical solutions and computational approaches to non-linear PDEs
- Analytical geometry
- Statistical Bayesian data assimilation of Earth-atmosphere-ocean observations

Predictability in a deterministic nonperiodic flow

"Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"
-(Lorenz 1972)





Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

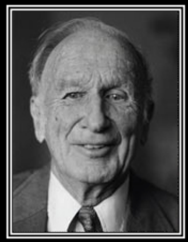
The feasibility of very-long-range weather prediction is examined in the light of these results.

Courtesy of A. Subramanian (Scipps)

Sensitive dependence to initial conditions

"Finite time for error in representation of small scales to affect accuracy of simulation of large scales, no matter how small in scale and hence amplitude this model error is"

-(Lorenz 1969)

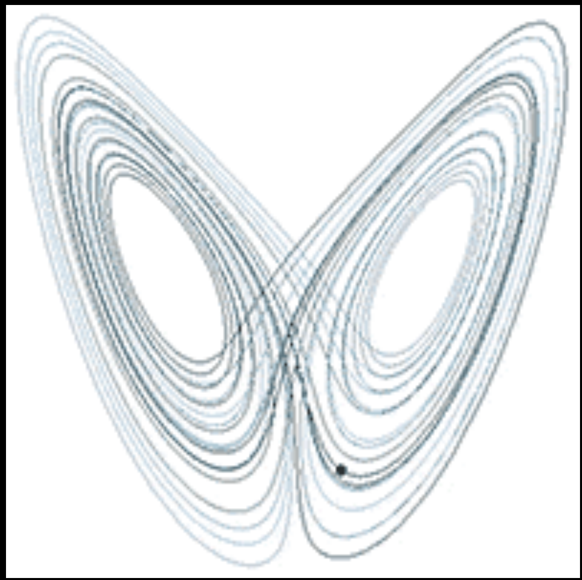


$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

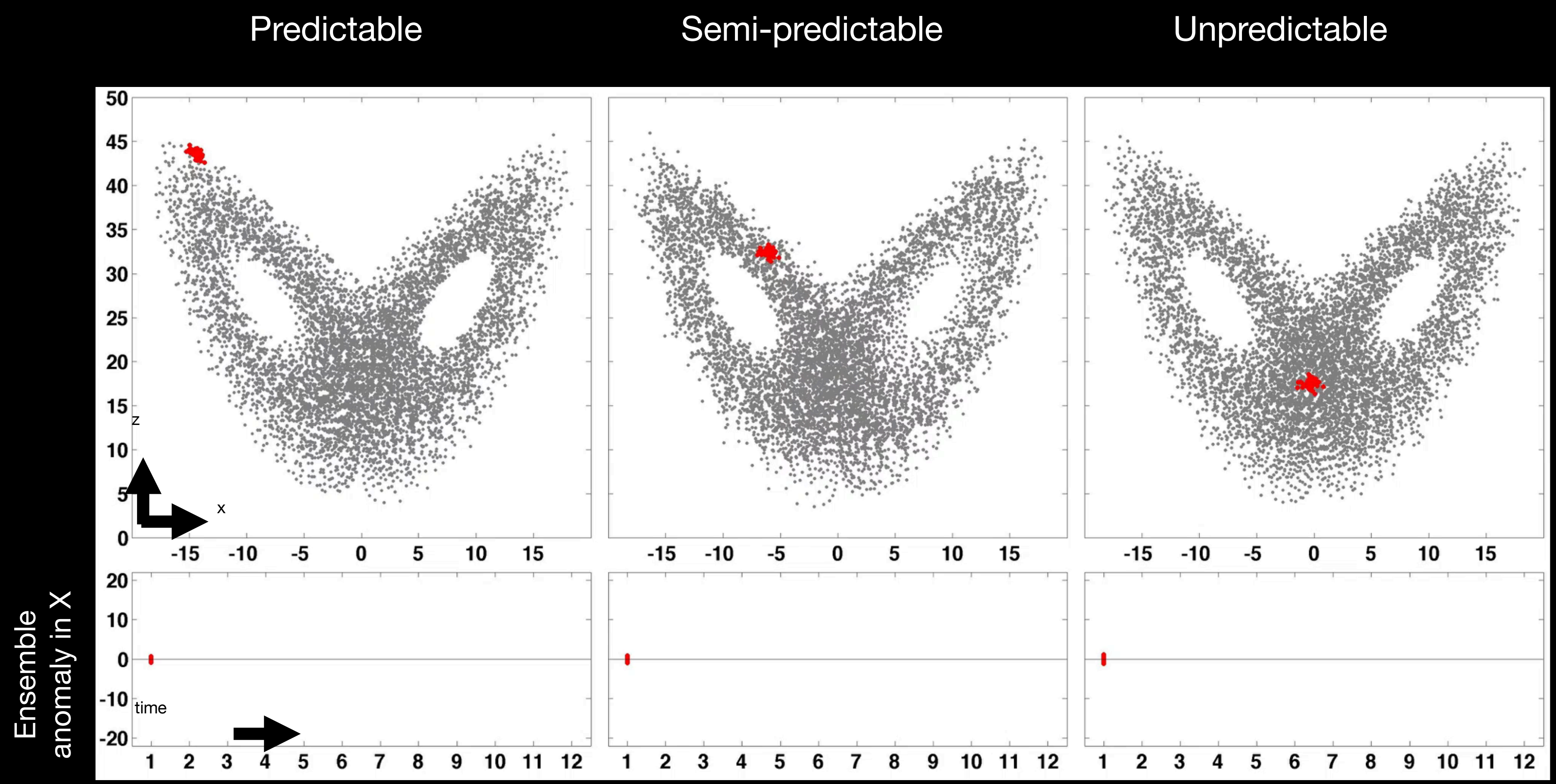
$$\frac{dz}{dt} = xy - bz$$

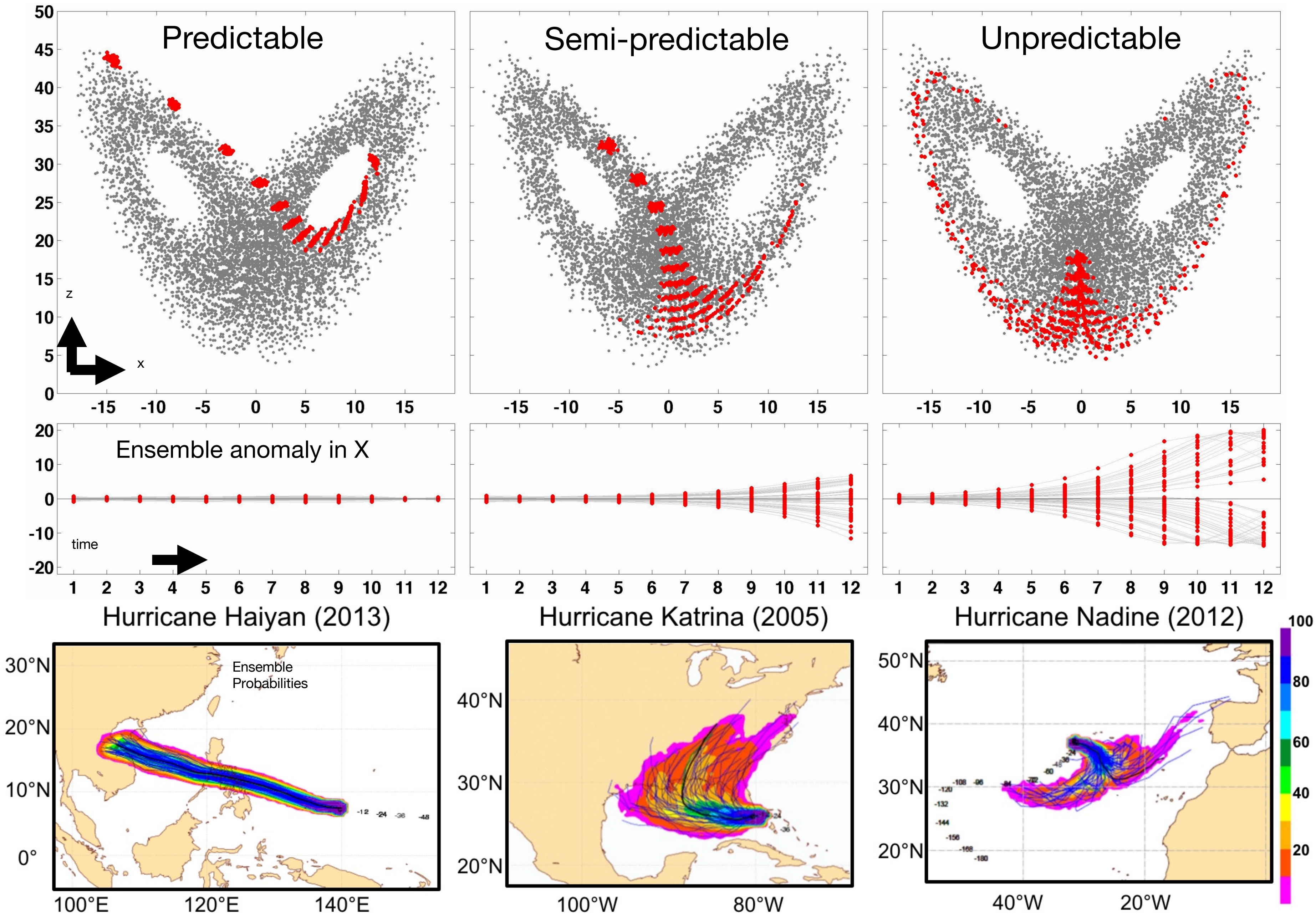
$r = 28$, $\sigma = 10$, and $b = 8/3$



source: wikipedia

Ensemble Forecast with Initial Uncertainty





Predictable
Semi-predictable
Unpredictable
z
x
Ensemble anomaly in X
time
Hurricane Haiyan (2013)
Hurricane Katrina (2005)
Hurricane Nadine (2012)
Ensemble
Probabilities
30°N
20°N
10°N
0°
100°E
120°E
140°E
40°N
30°N
20°N
100°W
80°W
50°N
40°N
30°N
20°N
40°W
20°W
100
80
60
40
20

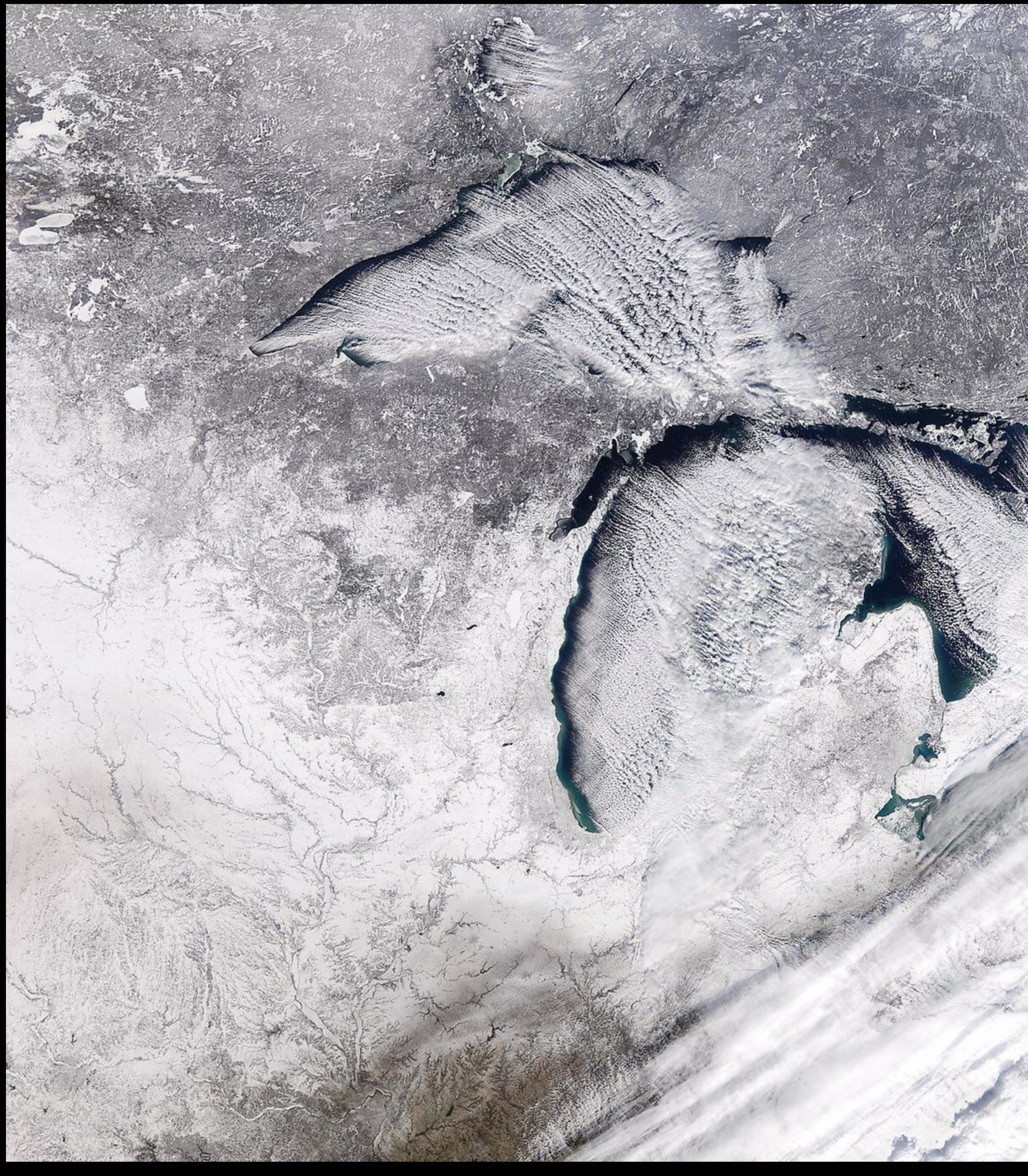
What's Really Warming the World?

Skeptics of manmade climate change offer various natural causes to explain why the Earth has warmed 1.4 degrees Fahrenheit since 1880. But can these account for the planet's rising temperature? Watch to see how much different factors, both natural and industrial, contribute to global warming, based on findings from NASA's Goddard Institute for Space Studies.

Bloomberg

Transition to Turbulence

Album of Fluid Motion (Van Dyke)

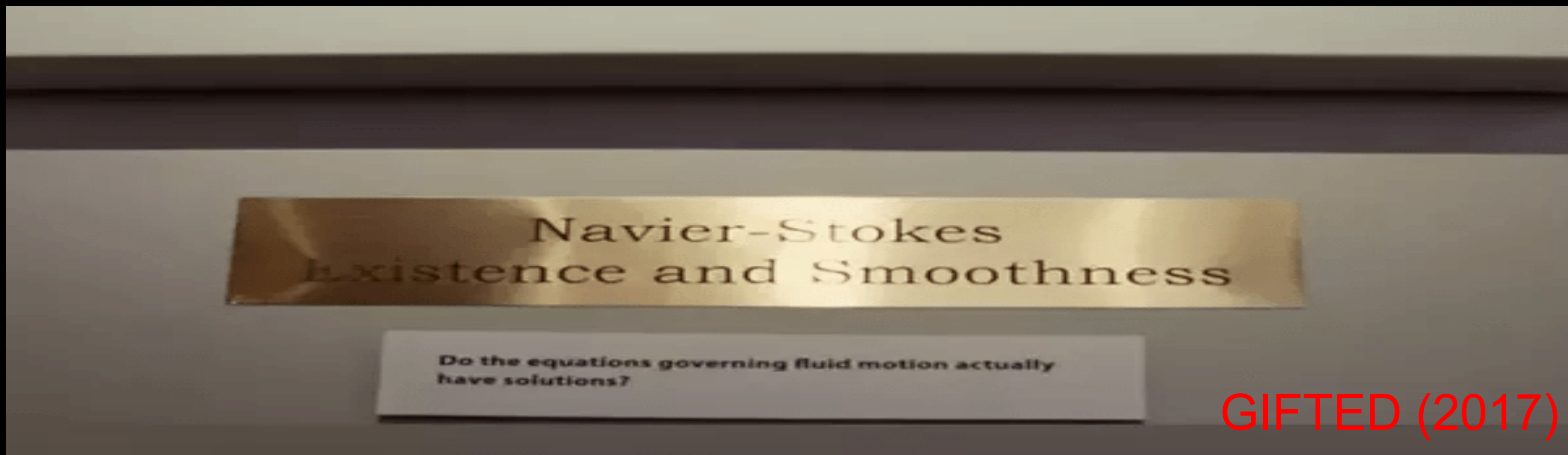


MODIS
Jan 2017

Navier-Stokes
a.k.a Newton's Second Law for a "Newtonian" Fluid

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}(\boldsymbol{x}, t)$$

N-S Smoothness and Existence Problem is a Millennial Math Problem



1. $\mathbf{v}(x,t) \in \left[C^\infty(\mathbb{R}^3 \times [0,\infty))\right]^3, \qquad p(x,t) \in C^\infty(\mathbb{R}^3 \times [0,\infty))$
2. There exists a constant $E \in (0,\infty)$ such that $\int_{\mathbb{R}^3} |\mathbf{v}(x,t)|^2 \, dx < E$ for all $t \geq 0$.

(A) Existence and smoothness of the Navier–Stokes solutions in $\mathbb{R}^3$

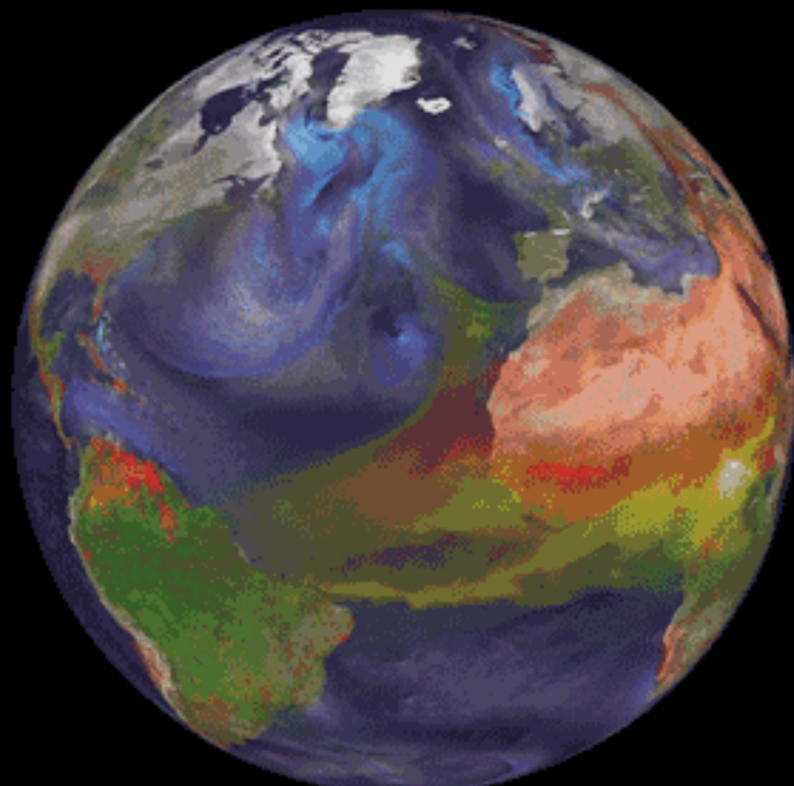
Let $\mathbf{f}(x,t) \equiv 0$. For any initial condition $\mathbf{v}_0(x)$ satisfying the above hypotheses there exist smooth and globally defined solutions to the Navier–Stokes equations, i.e. there is a velocity vector $\mathbf{v}(x,t)$ and a pressure $p(x,t)$ satisfying conditions 1 and 2 above.

(B) Breakdown of the Navier–Stokes solutions in $\mathbb{R}^3$

There exists an initial condition $\mathbf{v}_0(x)$ and an external force $\mathbf{f}(x,t)$ such that there exists no solutions $\mathbf{v}(x,t)$ and $p(x,t)$ satisfying conditions 1 and 2 above.

Going onto a rotating reference frame

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla \bar{p} + \mu \nabla^2 \mathbf{u} + \frac{1}{3}\mu \nabla(\nabla \cdot \mathbf{u}) + \rho \mathbf{g} - \rho \left(2\mathbf{\Omega} \times \mathbf{u} + \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{x} + \frac{d\mathbf{U}}{dt} + \frac{d\mathbf{\Omega}}{dt} \times \mathbf{x} \right).$$



N-S For Earth System

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho u_j}{\partial x_j},$$

$$\frac{\partial \theta}{\partial t} = -u_j \frac{\partial \theta}{\partial x_j} + S_\theta,$$

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} - \frac{1}{\rho}\frac{\partial p}{\partial x_i} - g\delta_{i3} - 2\epsilon_{ijk}\Omega_j u_k,$$

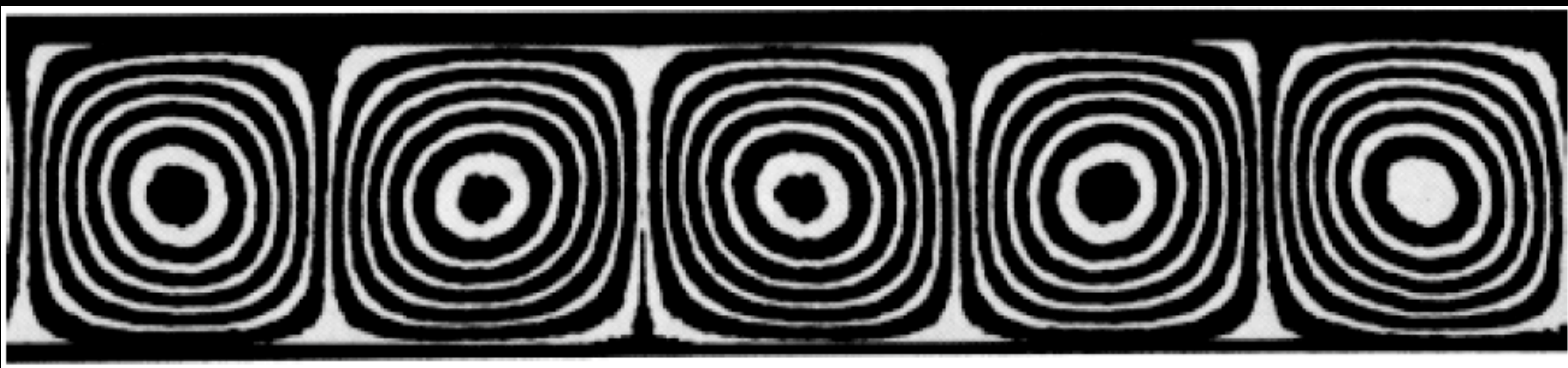
$$\frac{\partial q_{\mathrm{n}}}{\partial t} = -u_j \frac{\partial q_{\mathrm{n}}}{\partial x_j} + S_{q_n}, \qquad n = 1, 2, 3$$

One Solution to N-S: Convective instability

- Requires density to increase with height
- Instability occurs when Rayleigh number reaches critical threshold

$$\mathrm{Ra} = h^3 \Delta B / \nu\kappa > 1700$$

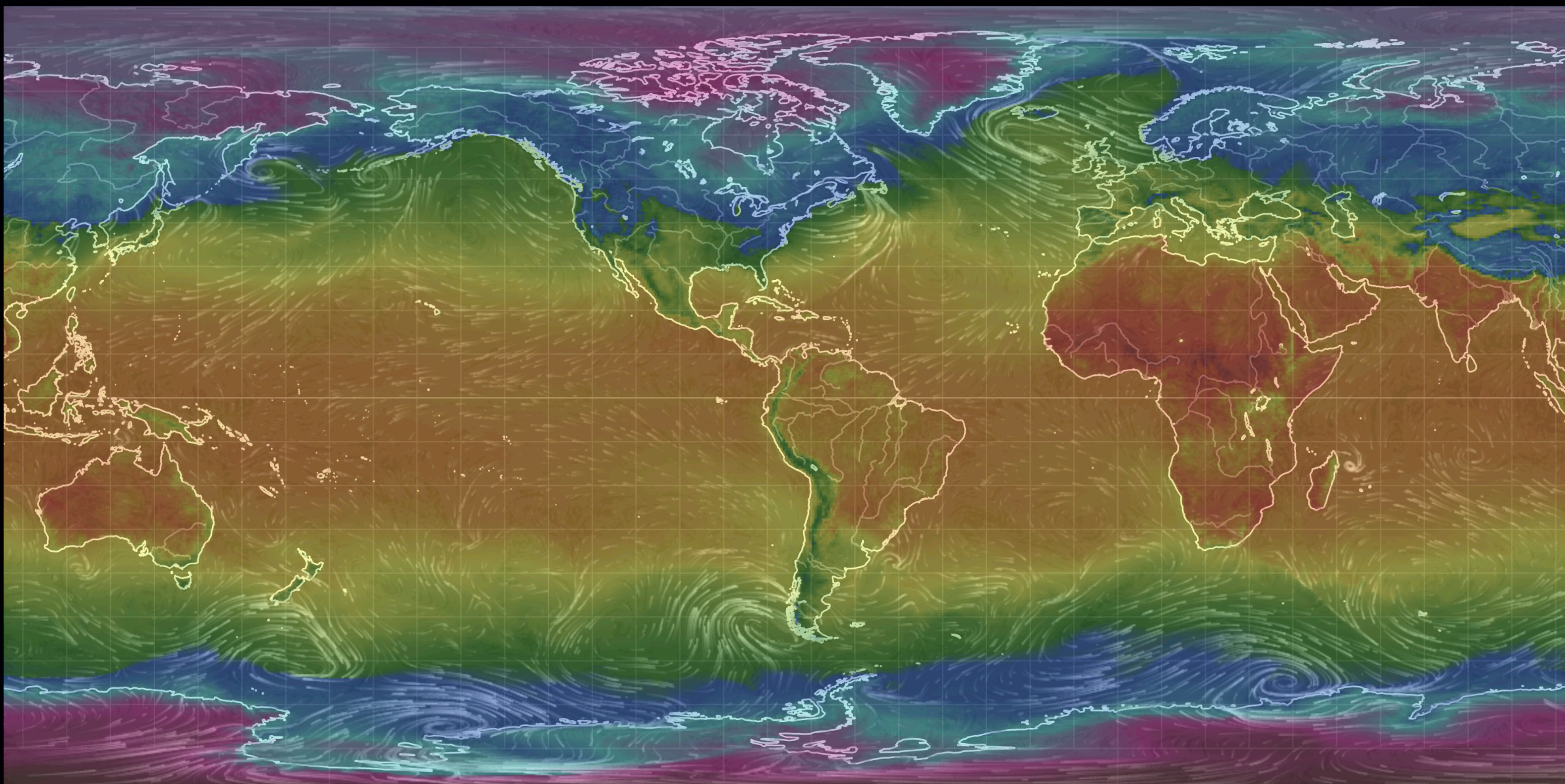
$$\Delta B = g \Delta T / T$$



Slightly unstable convection in silicone oil van Dyke p. 82

Cloud Streets: Convective Instability in the Real World

https://earth.nullschool.net/#current/wind/surface/level/overlay=temp/equirectangular



Turbulent Momentum Equation

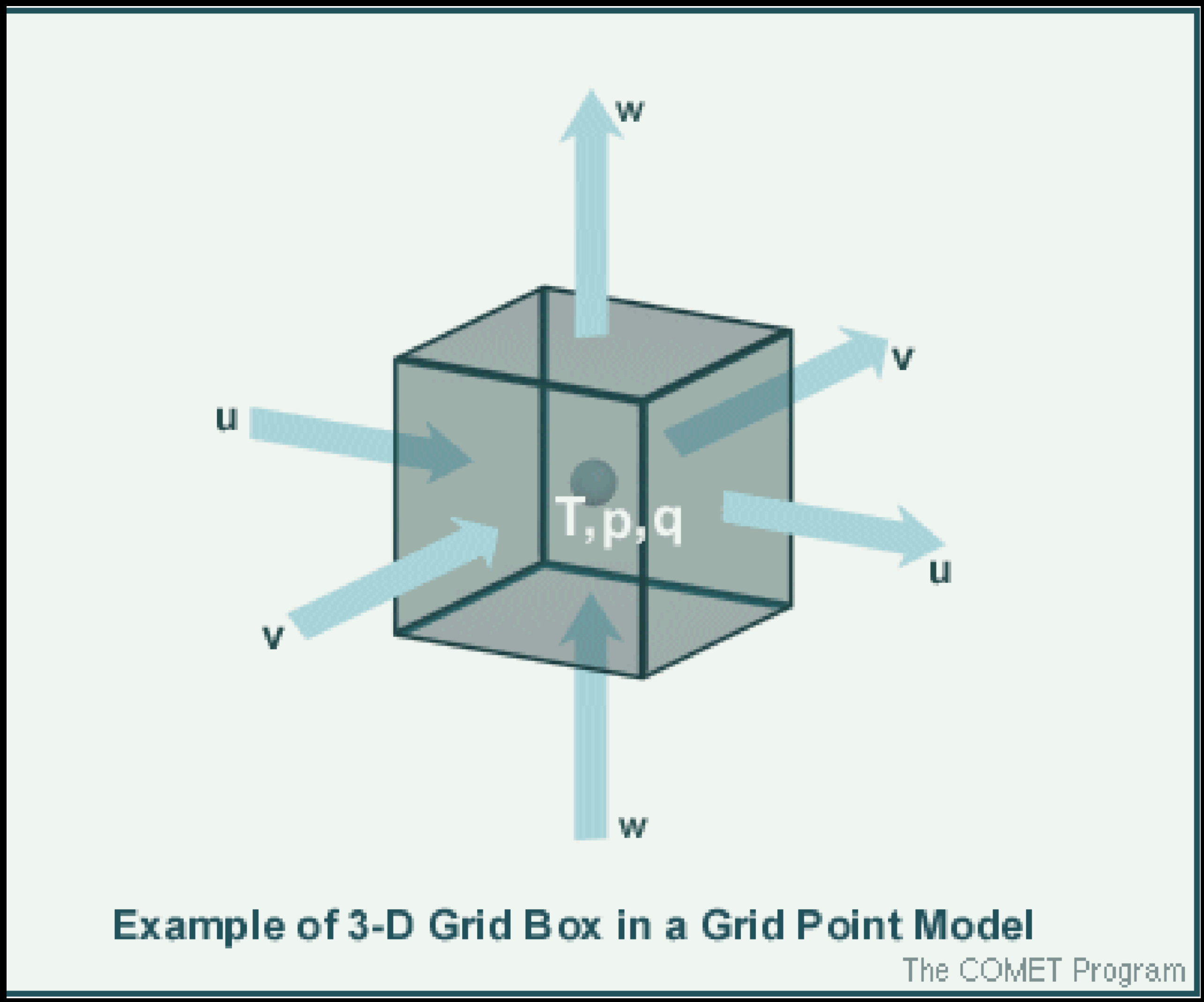
$$\frac{\partial \widetilde{U_i}}{\partial t} + \widetilde{U_j}\frac{\partial \widetilde{U_i}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widetilde{P}}{\partial x_i} - g\delta_{i3} + \frac{\mu}{\rho}\frac{\partial^2 \widetilde{U}_i}{\partial x_j^2}$$

…

$$\frac{\partial \overline{U_i}}{\partial t} + \overline{U_j}\frac{\partial \overline{U_i}}{\partial x_j} + \overline{u_j'\frac{\partial u_i'}{\partial x_j'}} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} - g\delta_{i3} + \frac{\mu}{\rho}\frac{\partial^2 \overline{U_i}}{\partial x_j^2}$$

$$\frac{\partial \overline{U_i}}{\partial t} + \frac{\partial\left[\overline{U_j U_i} + \overline{u_j' u_i'} - \nu\frac{\partial \overline{U_i}}{\partial x_j}\right]}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} - g\delta_{i3}$$

Numerical Solutions are Necessary



http://www

4 **Titan** - Cray XK7, C
NVIDIA K20x , Cray
DOE/SC/Oak Ridge
United States

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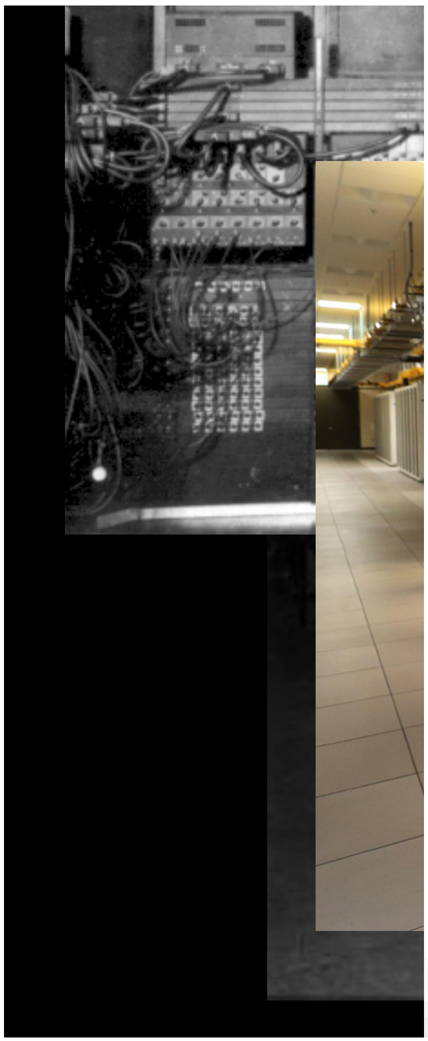

```
!**********************************************************************
      SUBROUTINE ADVE(NTSD,DT,DETA1,DETA2,PDTOP                        &
     &               ,CURV,F,FAD,F4D,EM_LOC,EMT_LOC,EN,ENT,DX,DY        &
     &               ,HTM,HBM2,VTM,VBM2,LMH,LMV                         &
     &               ,T,U,V,PDSLO,TOLD,UOLD,VOLD                        &
     &               ,PETDT,UPSTRM                                      &
     &               ,FEW,FNS,FNE,FSE                                   &
     &               ,ADT,ADU,ADV                                       &
     &               ,N_IUP_H,N_IUP_V                                   &
     &               ,N_IUP_ADH,N_IUP_ADV                               &
     &               ,IUP_H,IUP_V,IUP_ADH,IUP_ADV                       &
     &               ,IHE,IHW,IVE,IVW,INDX3_WRK                         &
     &               ,IDS,IDE,JDS,JDE,KDS,KDE                           &
     &               ,IMS,IME,JMS,JME,KMS,KME                           &
     &               ,ITS,ITE,JTS,JTE,KTS,KTE)
!**********************************************************************
!$$$  SUBPROGRAM DOCUMENTATION BLOCK
!                .      .    .
! SUBPROGRAM:    ADVE        HORIZONTAL AND VERTICAL ADVECTION
!   PRGRMMR: JANJIC          ORG: W/NP22     DATE: 93-10-28
!
! ABSTRACT:
!     ADVE CALCULATES THE CONTRIBUTION OF THE HORIZONTAL AND VERTICAL
!     ADVECTION TO THE TENDENCIES OF TEMPERATURE AND WIND AND THEN
!     UPDATES THOSE VARIABLES.
!     THE JANJIC ADVECTION SCHEME FOR THE ARAKAWA E GRID IS USED
!     FOR ALL VARIABLES INSIDE THE FIFTH ROW.  AN UPSTREAM SCHEME
!     IS USED ON ALL VARIABLES IN THE THIRD, FOURTH, AND FIFTH
!     OUTERMOST ROWS.  THE ADAMS-BASHFORTH TIME SCHEME IS USED.
!
! PROGRAM HISTORY LOG:
!   87-06-??  JANJIC     - ORIGINATOR
!   95-03-25  BLACK      - CONVERSION FROM 1-D TO 2-D IN HORIZONTAL
!   96-03-28  BLACK      - ADDED EXTERNAL EDGE
!   98-10-30  BLACK      - MODIFIED FOR DISTRIBUTED MEMORY
!   99-07-    JANJIC     - CONVERTED TO ADAMS-BASHFORTH SCHEME
!                          COMBINING HORIZONTAL AND VERTICAL ADVECTION
!   02-02-04  BLACK      - ADDED VERTICAL CFL CHECK
!   02-02-05  BLACK      - CONVERTED TO WRF FORMAT
!   02-08-29  MICHALAKES - CONDITIONAL COMPILATION OF MPI
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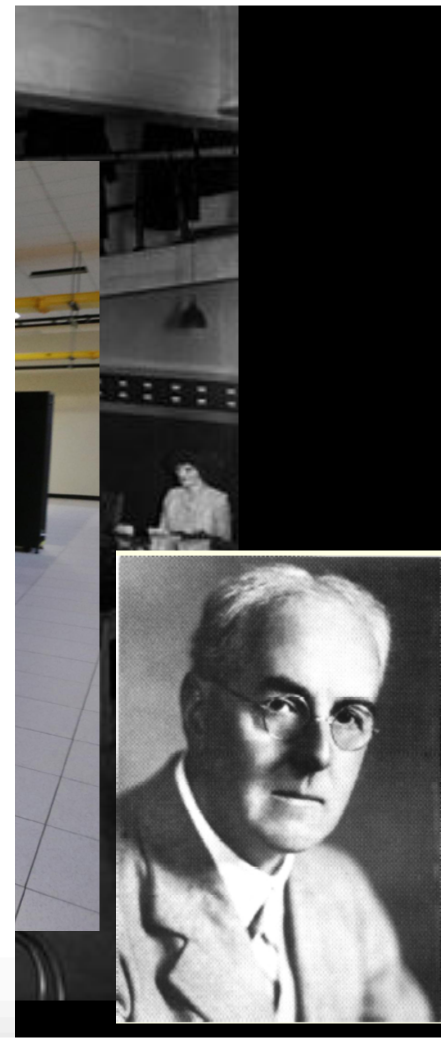
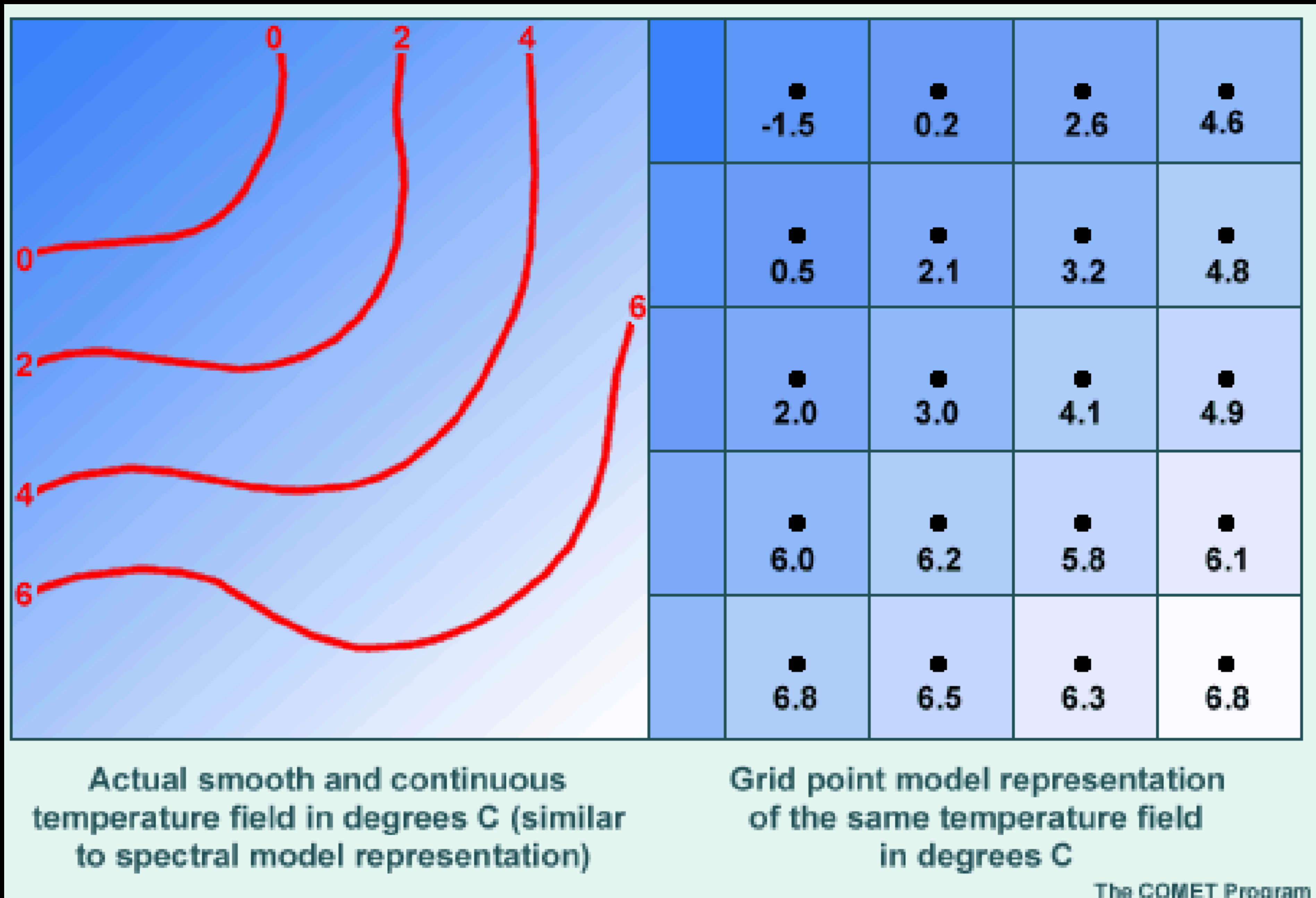
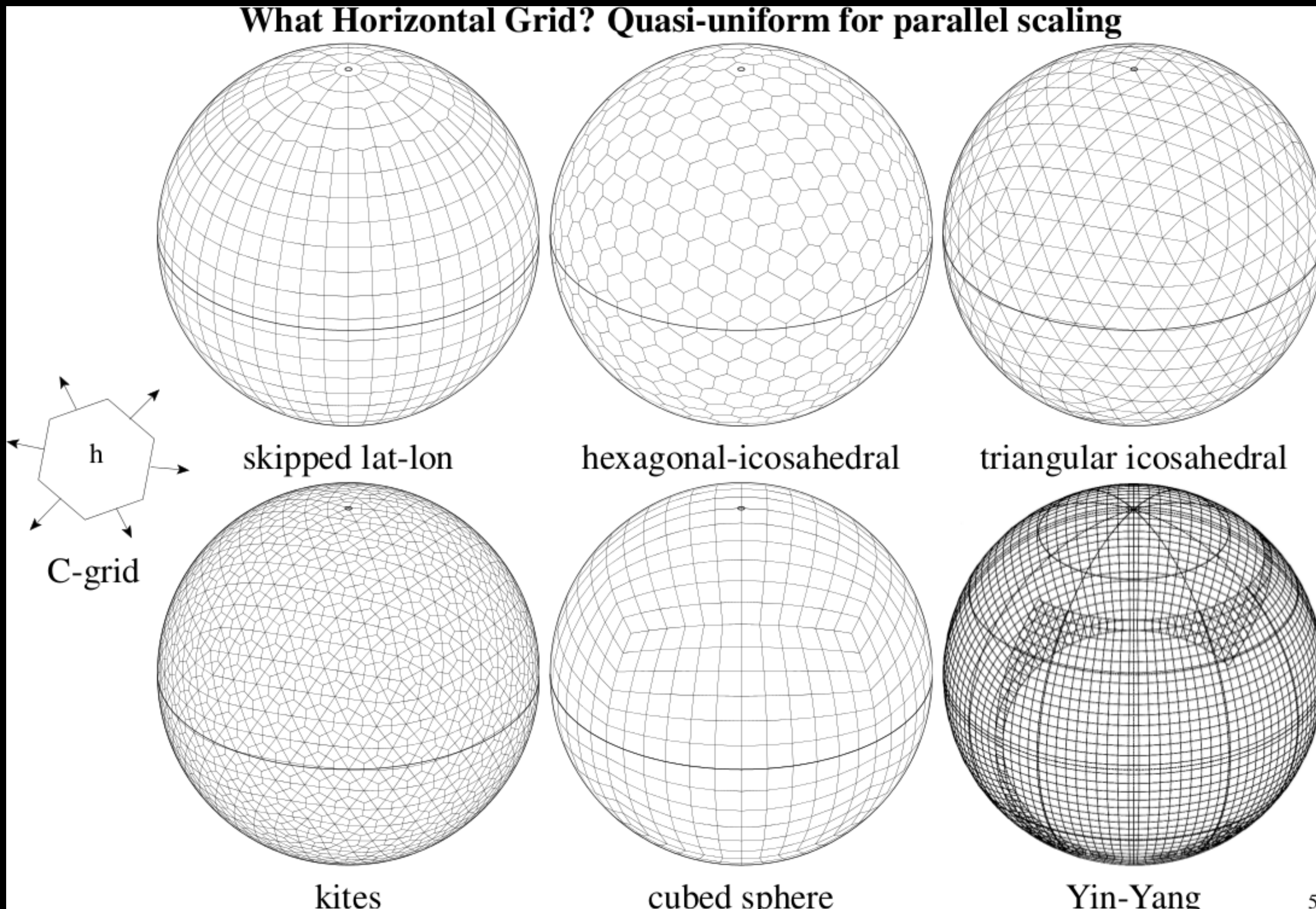
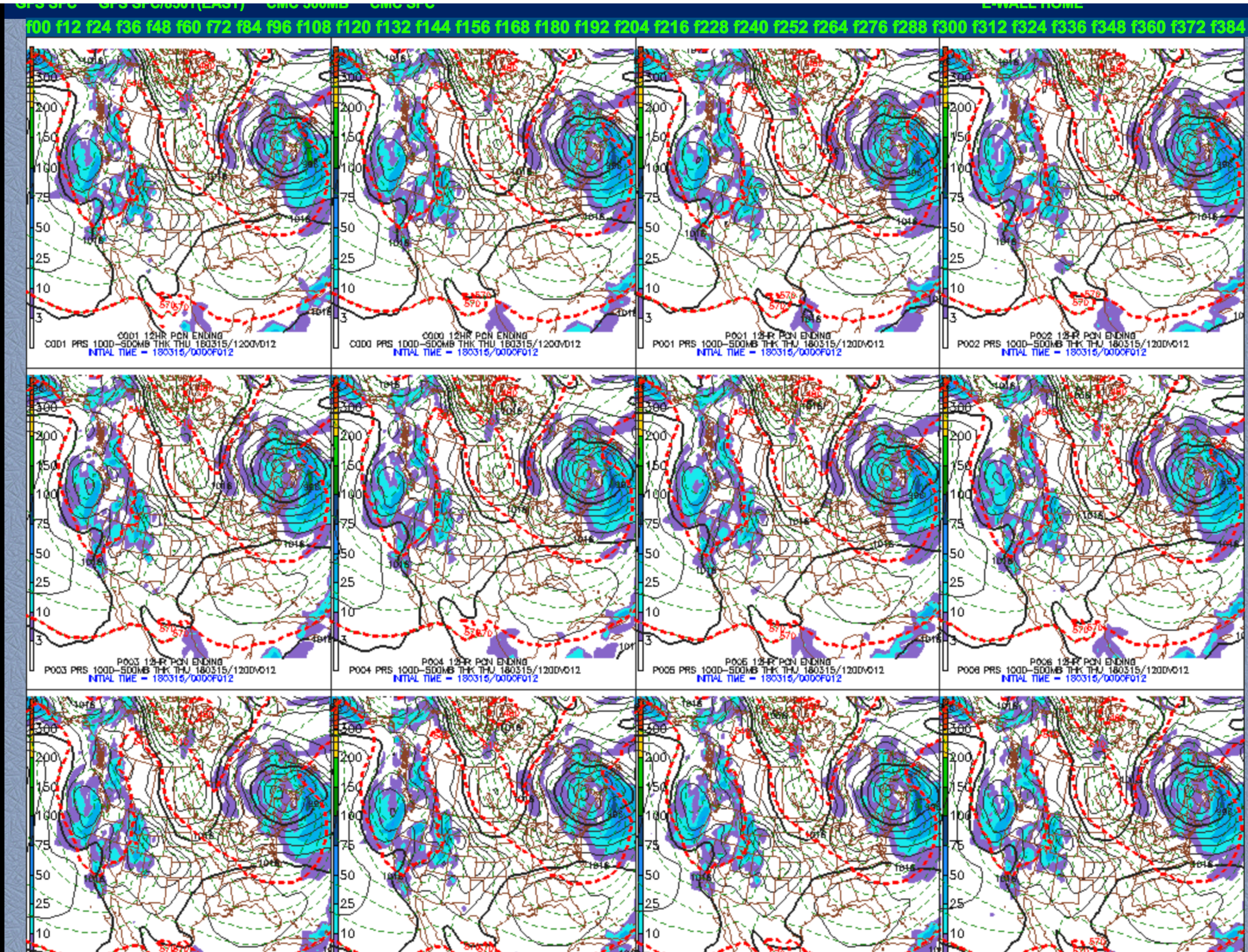



0
2
4
0
2
4
6
6
-1.5
0.2
2.6
4.6
0.5
2.1
3.2
4.8
2.0
3.0
4.1
4.9
6.0
6.2
5.8
6.1
6.8
6.5
6.3
6.8
Actual smooth and continuous temperature field in degrees C (similar to spectral model representation)
Grid point model representation of the same temperature field in degrees C
The COMET Program

But the Earth is a sphere

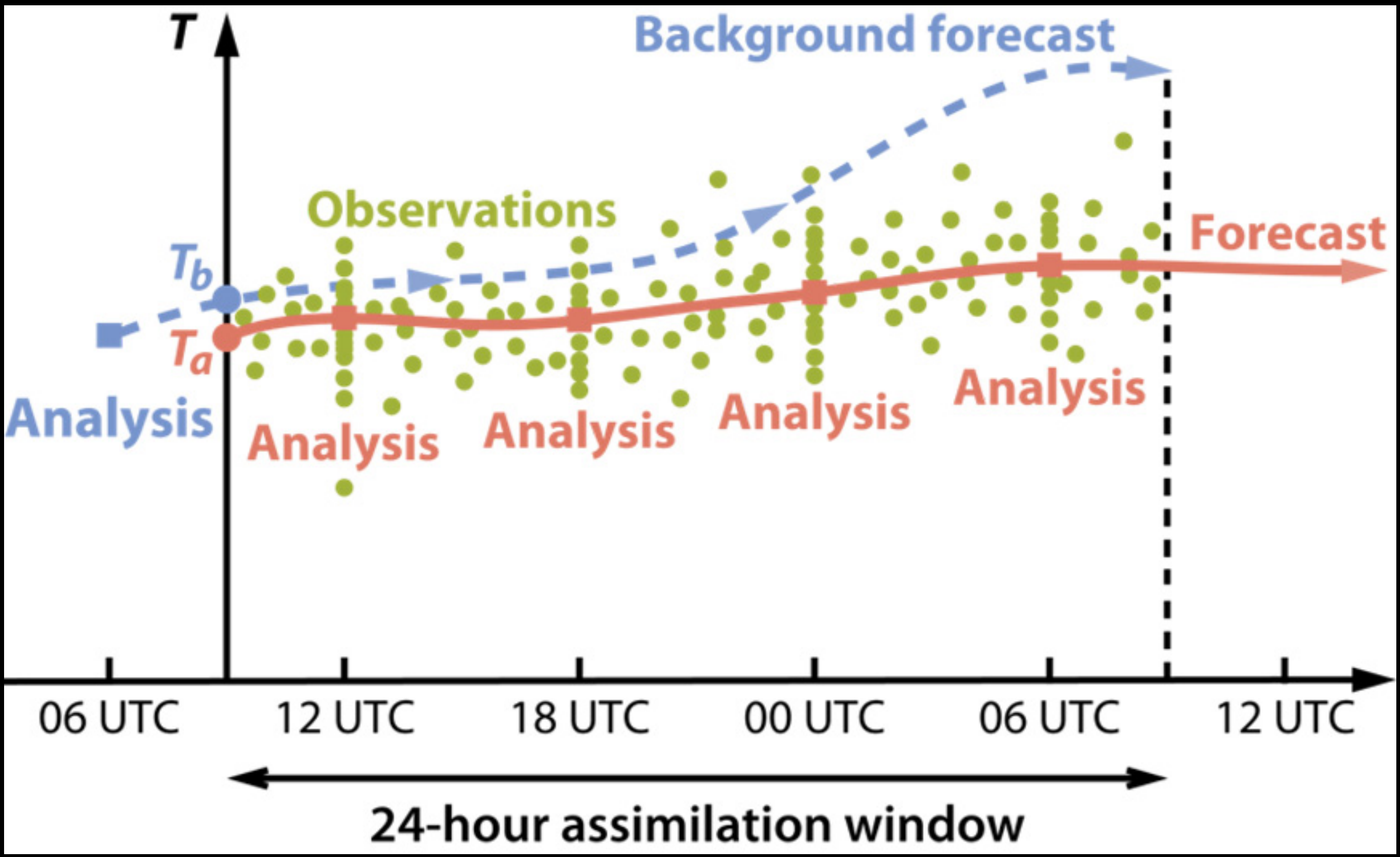


https://blogs.reading.ac.uk/wcd/2012/02/07/gungho-development-of-a-new-dynamical-core-for-the-unified-model/



http://mp1.met.psu.edu/~fxg1/ENSPRS_12z/ensloop.html

Model spread needs to be constrained by data



Applications of Bayes' Rule

Likelihood

How probable is the evidence given that our hypothesis is true?

Prior

How probable was our hypothesis *before* observing the evidence?

$$P(H \mid e) = \frac{P(e \mid H)\, P(H)}{P(e)}$$

Posterior

How probable is our hypothesis given the observed evidence? (Not directly computable)

Marginal

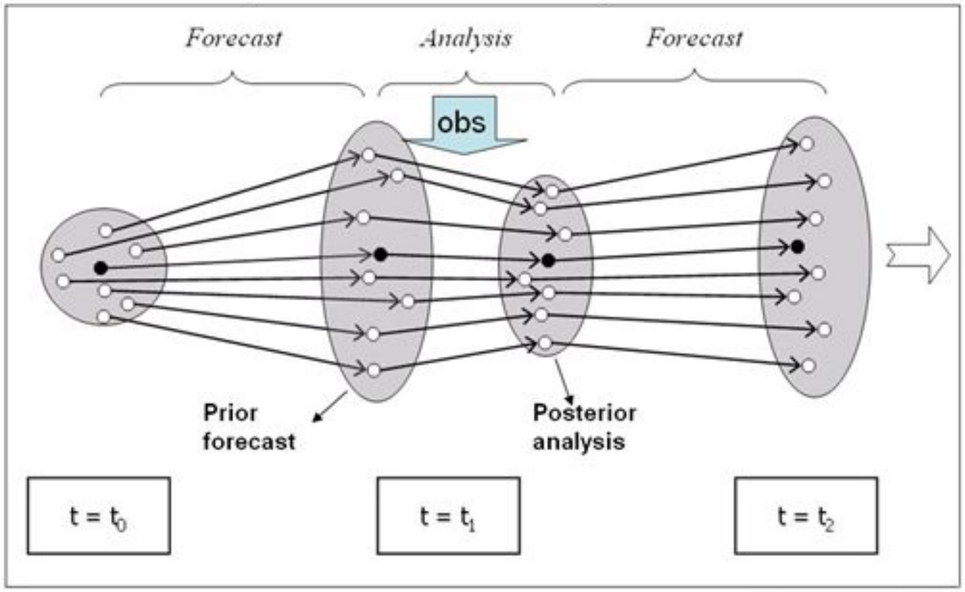
How probable is the new evidence under all possible hypotheses?

$P(e) = \sum P(e \mid H_i)\, P(H_i)$

Ensemble Kalman Filter

Ensemble Kalman filter is a Mont Carlo approximation of Kalman Filter. It samples the probability density function (PDF) of forecast and analysis using ensemble. (Evenson 1994).

$$X^a = X^f + K(y^o - HX^f)$$



(Aksoy 2003)

Before Ensemble Kalman filter

$$P^f \approx P_e^f = < x_i^{f'}, x_i^{f'} >$$

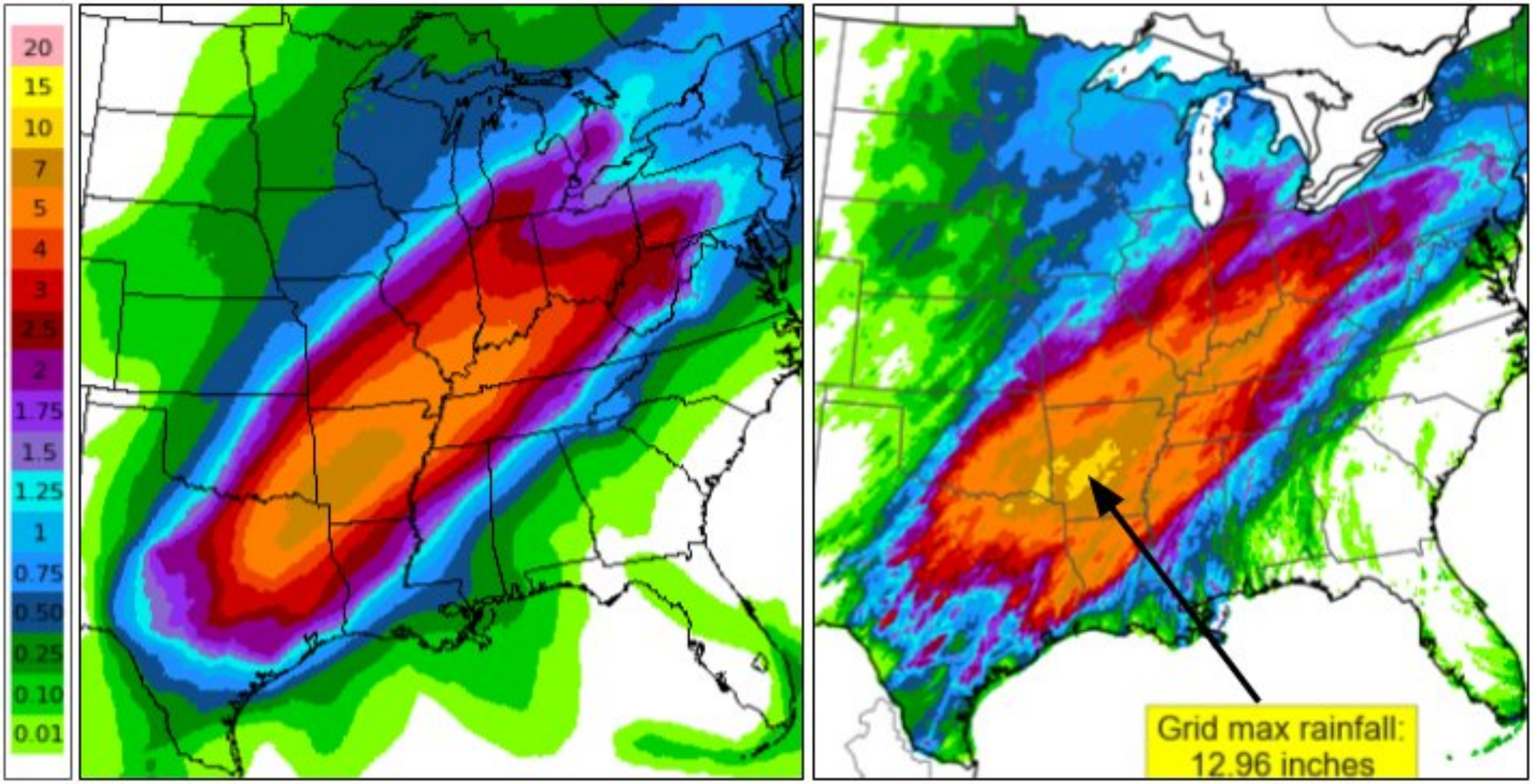
After Ensemble Kalman filter

$$P^a \approx P_e^a = < x_i^{a'}, x_i^{a'} >$$

http://slideplayer.com/slide/9781007/

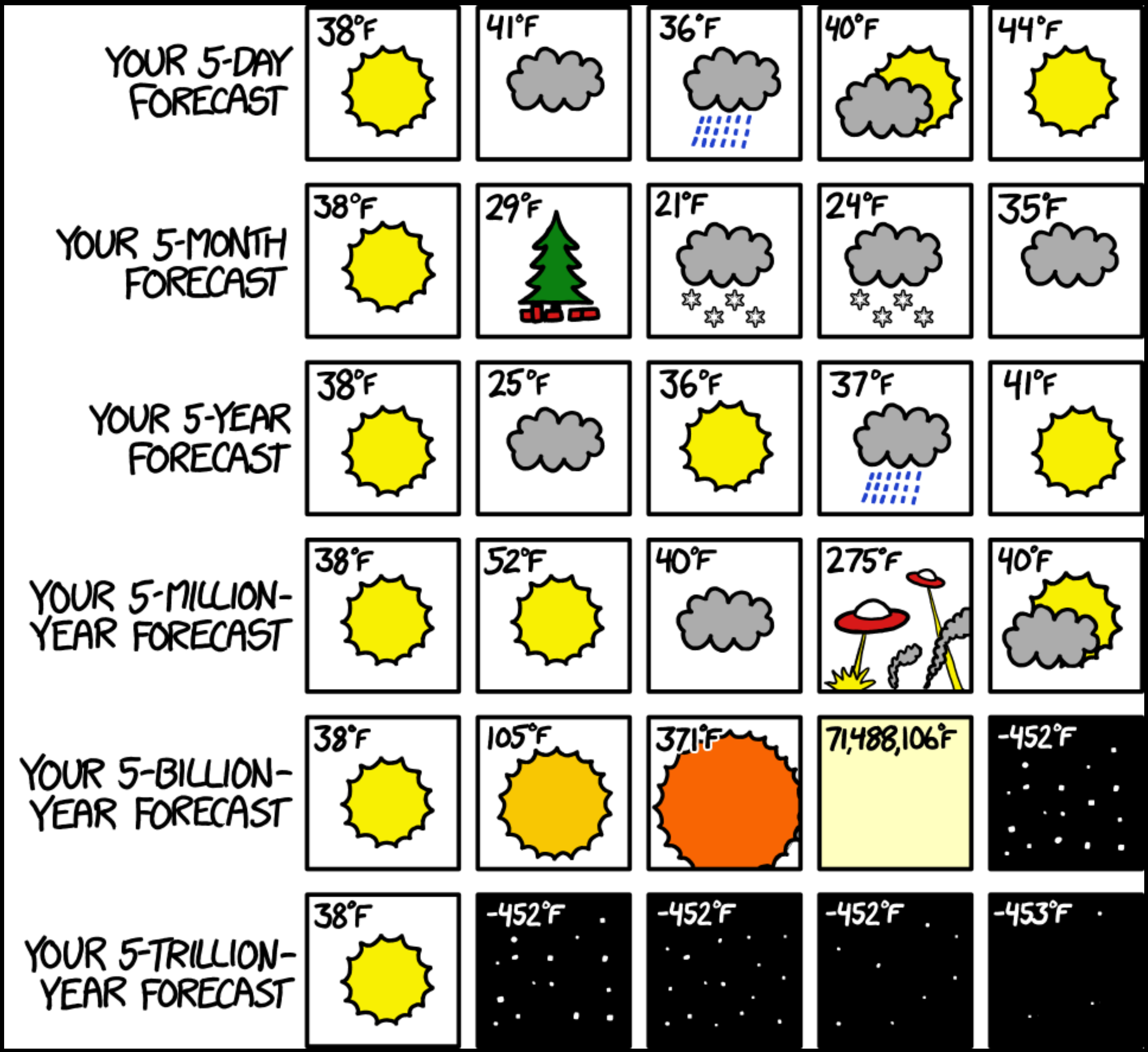
Stochastic filter and Deterministic filter

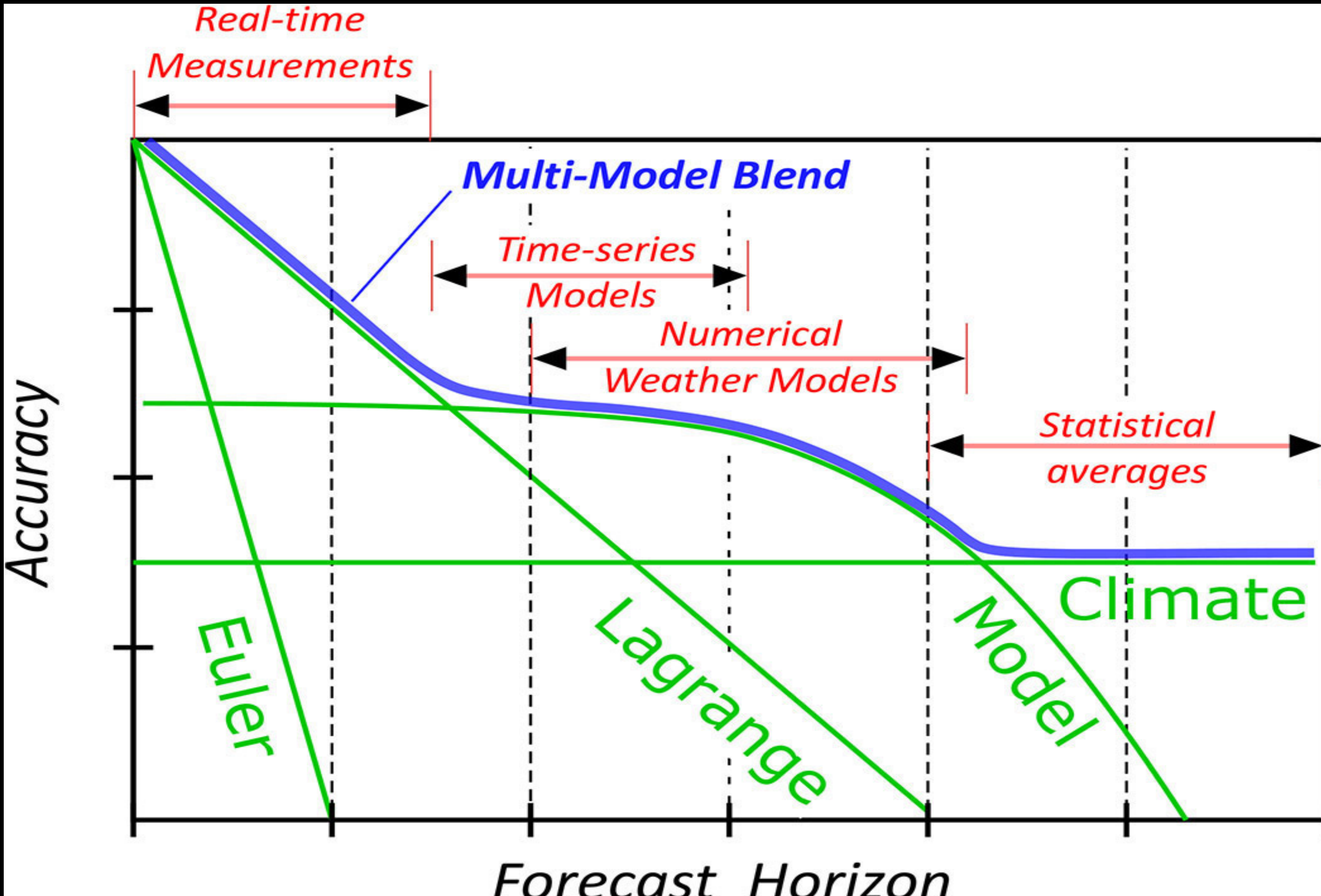
WPC 120h (5-day) Precipitation Forecast (left) vs. Observed Precipitation (NWS/AHPS, right)



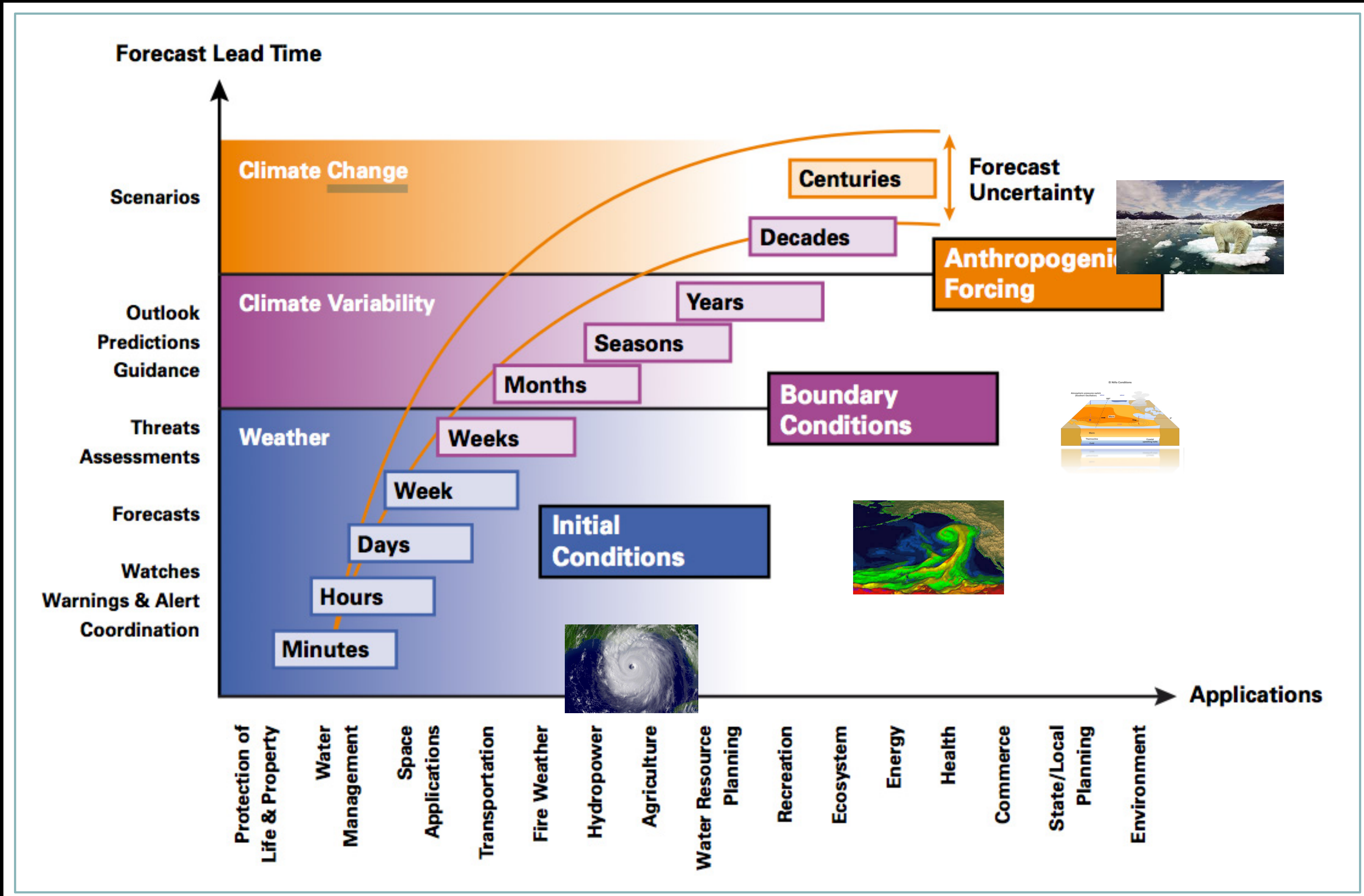
NWS/WPC 120h (5-day) forecast issued 0852 UTC Tue 2/20/18

NWS/AHPS 5-day total observed precipitation ending 12 UTC 2/25/18


YOUR 5-DAY FORECAST
38°F
41°F
36°F
40°F
44°F
YOUR 5-MONTH FORECAST
38°F
29°F
21°F
24°F
35°F
YOUR 5-YEAR FORECAST
38°F
25°F
36°F
37°F
41°F
YOUR 5-MILLION-YEAR FORECAST
38°F
52°F
40°F
275°F
40°F
YOUR 5-BILLION-YEAR FORECAST
38°F
105°F
371°F
71,488,106°F
-452°F
YOUR 5-TRILLION-YEAR FORECAST
38°F
-452°F
-452°F
-452°F
-453°F



http://spie.org/Images/Graphics/Newsroom/Imported-2015/006142/006142_10_fig1.jpg



source: https://www.wmo.int/

THANKS!

Ankur Desai, desai@aos.wisc.edu, @profdesai
+1-608-520-0305, http://flux.aos.wisc.edu

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